CONTROL OF LARGE-SCALE SYSTEMS:
BEYOND DECENTRALIZED FEEDBACK

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Abstract: In this paper, we present an array of new results that are aimed at broadening the scope of control design under information structure constraints. Both structural and algebraic enhancements of decentralized feedback will be considered, with convex optimization as a common mathematical framework. This approach leads to computationally efficient design strategies that are well suited for large-scale applications. In all cases, the obtained feedback laws guarantee robustness with respect to a wide range of nonlinear uncertainties, both within the subsystems and in the interconnections. 

Keywords: Dimensionality, decompositions, uncertainty, robustness, information structure constraints, output control, convex optimization.

1. INTRODUCTION

Decentralized control has been a control of choice for large-scale systems for over three decades. The reasons for the popularity of this approach are many, the most prominent being its ability to effectively solve problems of

- Dimensionality
- Uncertainty
- Information Structure Constraints

When a system has large dimension or consists of a large number of interconnected subsystems, it is computationally efficient to formulate control laws that use only locally available subsystem states or outputs. Such an approach is also economical, since it is easy to implement, and can significantly reduce costly communication overhead. Robustness is another attractive feature of decentralized control laws, since they can make stability of the closed-loop system tolerant to a broad range of uncertainties, both within the subsystems and in their interconnections. Finally, when exchange of state information among the subsystems is prohibited, decentralized structure becomes an essential design constraint. The vast body of existing literature on the decentralized control of large-scale systems (including both theory and applications) has been reviewed in a number of survey papers and books (Ikeda, 1989; Tamura and Yoshikawa, 1990; Šiljak, 1991; Sezer and Šiljak, 1996; Šiljak, 1996; Jamshidi, 1997; Šiljak and Zečević, 1999; Wu 2003).

A widespread extension of decentralized control arises in applications where some state information is shared among the subsystems. When a system contains overlapping subsystems, it is natural to add the locally available overlapping states to decentralized control in order to improve the overall performance. Control design with overlapping information structure is typically handled by expanding the system into a larger space where the overlapping subsystems appear as disjoint. As a result of the expansion, overlapping decentralized control in the expanded space can be obtained by standard methods that are available for disjoint subsystems. After the selection is made, the expanded control law is contracted to the original space for implementation. The mathematical framework for the expansion-contraction process has been formulated as the Inclusion Principle (Ikeda et al., 1984; for a self-contained presentation of the early results, see Šiljak, 1991).

The objective of this paper is to present new methods for broadening the scope of control design under
information structure constraints. Both structural and algebraic enhancements of decentralized control will be proposed. One of our key contributions in the context of structural enhancements is a new approach to the problem of contractibility in overlapping decentralized structures. The central issue in the design of overlapping decentralized control has been the problem of contracting stabilizing feedback laws from the expanded space to the original space, which is essential for implementation purposes (Ikeda et al., 1981, 1986; Iftar and Özgüner, 1990; Iftar, 1993a,b; Stanković and Šiljak, 2001, 2003; Bakule et al., 2003; Stanković, 2004). Despite a large number of contractibility conditions, the general resolution of this problem remains an open question. With that in mind, in Section 2 we will describe a new structural approach to contractibility of control laws within the framework of convex optimization.

In applications where communication overhead is a concern, it is desirable to utilize structures that offer maximal improvement in performance at a minimal cost in information exchange. For this purpose, in Section 2 we will introduce a new form of gain matrices that are characterized by a Border-Block Diagonal (BBD) structure. Such a structure can significantly improve the decentralized stabilization of large systems, at the expense of only minimal communication overhead.

Sections 3 and 4 are devoted to alternative algebraic results for enhancing decentralized control. These include new methods for the design of decentralized static output feedback, low-rank augmentation of decentralized control gains, and a new scheme for reduced-order controller design. In all cases, the design objective can be formulated in terms of a special factorization of the gain matrix. We will show how such constraints can be incorporated into the framework of convex optimization, and how this approach leads to computationally efficient design strategies. Needless to say, this feature is of considerable importance for large-scale applications, where computational complexity is a major concern.

Throughout the paper, our analysis will focus on nonlinear systems of the form

\[
\dot{x} = Ax + h(x) + Bu
\]

\[
y = Cx
\]  

(1)

where \(A, B\) and \(C\) are constant \(n \times n\), \(n \times m\) and \(q \times n\) matrices, and \(h: \mathbb{R}^n \to \mathbb{R}^q\) is a piecewise-continuous nonlinear function in \(x\), satisfying \(h(0) = 0\). The term \(h(x)\) may contain uncertainties, but we assume that it can be bounded by a quadratic inequality

\[
h^T(x)h(x) \leq \alpha^2 x^T H^T H x
\]

(2)

where \(H\) is a constant matrix, and \(\alpha > 0\) is a scalar parameter. In cases when the system is input/output decentralized, we will use the notation \(B_D\) and \(C_D\) to indicate that matrices \(B_D = \text{diag}\{B_1, \ldots, B_N\}\) and \(C_D = \text{diag}\{C_1, \ldots, C_N\}\) consist of \(n_i \times m_i\) and \(q_i \times n_i\) diagonal blocks, respectively.

2. STRUCTURAL CONSTRAINTS ON THE GAIN MATRIX

From the standpoint of control design, information structure constraints can be distinguished by the type of restrictions that they place on the gain matrix. The most obvious restrictions are those that are structural in nature. Several generic scenarios are shown in Figs. 1-3, corresponding to decentralized, overlapping and BBD state feedback, respectively. It is important to note that all three structures can be obtained using linear matrix inequalities (LMI) (Boyd, et al., 1994; Geromel et al., 1994; El Ghaoui and Niculescu, 2000) and the general mathematical framework proposed in (Šiljak and Stipanović, 2000; Šiljak et al., 2002; Zečević and Šiljak, 2003). The sufficient conditions and algorithms for such a design are described in the remainder of this section.

![Fig. 1. A block diagonal gain matrix structure.](image1)

\[
K = \begin{bmatrix}
K_{11} & 0 & 0 \\
0 & K_{22} & 0 \\
0 & 0 & K_{33}
\end{bmatrix}
\]

![Fig. 2. An overlapping gain matrix structure.](image2)

\[
K = \begin{bmatrix}
K_{11} & K_{12} & 0 & 0 \\
0 & K_{22} & K_{23} & 0 \\
0 & 0 & K_{33}
\end{bmatrix}
\]

![Fig. 3. A BBD gain matrix structure.](image3)

\[
K = \begin{bmatrix}
K_{11} & 0 & K_{13} \\
0 & K_{22} & K_{23} \\
K_{31} & K_{32} & K_{33}
\end{bmatrix}
\]
2.1 Decentralized and BBD State Feedback

In the case of decentralized state feedback, the control design amounts to solving the following optimization problem in $\gamma$, $\kappa_Y$, $\kappa_L$, $Y$ and $L$ (Šiljak and Stipanović, 2000).

**Problem 1.** Minimize $a_1\gamma + a_2\kappa_Y + a_3\kappa_L$ subject to

$$
Y > 0
$$

$$
\begin{bmatrix}
AY + YA^T + BL + L^TB^T & I & YH^T \\
I & -I & 0 \\
HY & 0 & -\gamma I
\end{bmatrix} < 0
$$

and

$$
\gamma^{-1/\alpha^2} < 0
$$

Several comments need to be made regarding this procedure.

**Remark 1.** The gain matrix is computed as $K = LY^{-1}$, and its norm is implicitly constrained by inequalities (6), which imply that $\|K\| \leq \sqrt{\kappa_L \kappa_Y}$. This is necessary in order to prevent unacceptably high gains that an unconstrained optimization may otherwise produce.

**Remark 2.** Matrices $L$ and $Y$ must both be block-diagonal, with blocks of dimension $m_i \times n_i$ and $n_i \times n_i$, respectively. In that case, matrix $K = LY^{-1}$ is guaranteed to have the same structure as $L$.

**Remark 3.** If the optimization problem (3)-(6) is feasible, the resulting gain matrix $K$ stabilizes the closed-loop system for all nonlinearities satisfying (2). Condition (5) additionally secures that $\alpha$ is greater than some desired robustness bound $\alpha$.

**Remark 4.** The proposed design procedure can also be used to obtain a bordered block-diagonal gain matrix. The only necessary modification in this case is to ensure that matrix $L$ has a BBD structure identical to that of matrix $K$.

**Remark 5.** The special structure of matrices $L$ and $Y$ substantially reduces the number of LMI variables. As a result, the proposed optimization is well suited for large-scale systems.

2.2 Overlapping State Feedback

The design of overlapping control is somewhat more complicated, and its treatment depends to a large extent on the structure of matrix $B$. Assuming that $B$ has the generic form

$$
B = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \\ 0 & B_{32} \end{bmatrix}
$$

two possible scenarios can arise. These scenarios are schematically represented in Figs. 4 and 5, with Type II corresponding to the situation when $B_{21} = 0$ and $B_{22} = 0$.

In the case of Type I overlapping, we can use optimization Problem 1 directly, with matrices $L$ and $Y$ in the form

$$
L = \begin{bmatrix} L_{11} & L_{12} & 0 \\ 0 & L_{22} & L_{23} \end{bmatrix}, \quad Y = \begin{bmatrix} Y_{11} & 0 & 0 \\ 0 & Y_{22} & 0 \\ 0 & 0 & Y_{33} \end{bmatrix}
$$

since this implies

$$
K = LY^{-1} = \begin{bmatrix} L_{11}Y_{11}^{-1} & L_{12}Y_{22}^{-1} & 0 \\ 0 & L_{22}Y_{22}^{-1} & L_{23}Y_{33}^{-1} \end{bmatrix}
$$

For Type II overlapping, however, this strategy usually leads to infeasibility, and alternative methods must be explored.

Fig. 4. A schematic representation of Type I overlapping.
Fig. 5. A schematic representation of Type II overlapping.

One possible approach is based on the expansion of system

$$S: \quad \dot{x} = Ax + Bu$$

as

$$\tilde{S}: \quad \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u$$

where $\tilde{A}$ and $\tilde{B}$ are of dimension $\tilde{n} \times \tilde{n}$ and $\tilde{n} \times m$, respectively (with $\tilde{n} > n$). The two systems are related by

$$VA = \tilde{A}V; \quad VB = \tilde{B}$$

where $V$ is a rectangular $\tilde{n} \times n$ transformation matrix.

It is well known that if the expanded system $\tilde{S}$ can be stabilized with a decentralized control $u = \tilde{K}_D\tilde{x}$, where

$$\tilde{K}_D = \begin{bmatrix} \hat{K}_{11} & \hat{K}_{12} & 0 & 0 \\ 0 & 0 & \hat{K}_{23} & \hat{K}_{24} \end{bmatrix}$$

then the original system $S$ is stabilized by $u = Kx$, where

$$K = \tilde{K}_DV = \begin{bmatrix} \hat{K}_{11} & \hat{K}_{12} & 0 \\ 0 & 0 & \hat{K}_{23} & \hat{K}_{24} \end{bmatrix}$$

represents an overlapping control law.

Since the expanded system $\tilde{S}$ is known to be inherently uncontrollable (see e.g. Šiljak, 1991), our objective in this case will be to determine a gain matrix $\tilde{K}_D$ and a compatible block-diagonal matrix $\tilde{P}_D > 0$ such that inequality

$$V^T \left( (A + B\tilde{K}_D)^T \tilde{P}_D + \tilde{P}_D(A + B\tilde{K}_D) \right) V < 0$$

holds. If such matrices exist, it is easily verified that (15) implies

$$(A + BK)^T W + W(A + BK) < 0$$

where $W = V^T \tilde{P}_DV$. The stability of the closed-loop system is therefore guaranteed whenever (15) is satisfied, and $v(x) = x^T W x$ represents an appropriate Lyapunov function.

In order to formulate condition (15) in the LMI framework, we should first observe that it is a nonlinear matrix inequality in $\tilde{P}_D$ and $\tilde{K}_D$. To avoid this difficulty, we will introduce a fixed $\tilde{n} \times n$ matrix $G$ of full rank, and assume that there exists a nonsingular $n \times n$ matrix $M$ such that

$$\tilde{P}_D VM = GM$$

Defining $\tilde{Y}_D = \tilde{P}_D^{-1}$ and $\tilde{L}_D = \tilde{K}_D\tilde{Y}_D$, (15) can now be equivalently rewritten as

$$G^T (\tilde{Y}_D \tilde{A}^T + \tilde{A}\tilde{Y}_D + \tilde{B}\tilde{L}_D + \tilde{L}_D^T \tilde{B}^T) G < 0$$

which is an LMI in $\tilde{Y}_D$ and $\tilde{L}_D$. The construction of an appropriate matrix $G$ and extensions to nonlinear systems of the form (1) have been studied in (Zečević and Šiljak, 2004a), and will not be discussed here.

### 3. Algebraic Constraints on the Gain Matrix

Algebraic constraints require matrix $K$ to be factorizable in a special form. In the following, we will consider two cases that are of particular interest: the design of static output feedback and decentralized control with a global low-rank enhancement.

#### 3.1 Static Output Feedback

For the purposes of static output feedback, the closed-loop system (1) must have the form

$$\dot{x} = (A + BK)x + h(x)$$

With that in mind, our main objective in the following will be to modify the LMI optimization (3)-(6) so that the product $LY^{-1}$ can be factorized as
\[ LY^{-1} = KC \quad (20) \]

A simple approach to this problem would be to look for a solution in which matrix \( L \) has the form

\[ L = L_C U^T \quad (21) \]

where \( U \) is a fixed \( n \times q \) matrix and \( L_C \) is an unknown \( m \times q \) matrix. In that case, (20) holds whenever

\[ U^T Y^{-1} = C \quad (22) \]

with \( K = L_C \).

Condition (22) can be included in the LMI optimization by adding the equality constraint

\[ YC^T = U \quad (23) \]

In this context, we should note that (23) is automatically satisfied if we set \( U = C^T \), and look for \( Y \) in the form

\[ Y = QY_Q Q^T + C^T (CC^T)^{-1} C \quad (24) \]

where \( Q \) is an \( n \times (n-q) \) matrix such that

\[ Q^T C^T = 0 \quad (25) \]

and \( Y_Q \) is an unknown symmetric matrix of dimension \((n-q) \times (n-q)\). Under such circumstances, the LMI optimization does not require an explicit equality constraint, but the number of LMI variables associated with \( Y \) is reduced from \( n(n+1)/2 \) to \( (n-q)(n-q+1)/2 \).

The reduction of variables due to (24) can have a detrimental effect on the feasibility of the optimization, particularly in cases where \( q \) is relatively large. For this reason, we propose to introduce additional LMI variables by looking for a solution of Problem 1 in the form

\[ Y = Y_0 + UY_C U^T \quad (26) \]

\[ L = L_C U^T \]

where \( Y_0 \) and \( Y_C \) are unknown symmetric matrices of dimensions \( n \times n \) and \( q \times q \), respectively. For any given choice of \( U \), the optimization (3)-(6) then becomes an LMI problem in \( \gamma, \kappa_Y, \kappa_L, Y_0, Y_C \), and \( L_C \).

To see the connection between (26) and the desired output feedback structure, we should observe that \( Y^{-1} \) can be expressed using the Sherman-Morrison formula as (e.g. Golub and van Loan, 1996)

\[ Y^{-1} = Y_0^{-1} - S R U^T Y_0^{-1} \quad (27) \]

where

\[ S = Y_0^{-1} UY_C \]
\[ R = [I + U^T S]^{-1} \quad (28) \]

It is now easily verified that condition

\[ Y_0 C^T = U \quad (29) \]

ensures that \( LY^{-1} = KC \), with

\[ K = L_C (I - U^T SR) \quad (30) \]

As before, the equality constraint (29) can be automatically satisfied if we set \( U = C^T \), and look for \( Y_0 \) in the form (24). We should note that the overall number of LMI variables associated with \( Y \) is now increased by \( q(q+1)/2 \), due to the presence of matrix \( Y_C \). The corresponding design procedure is a relatively simple modification of Problem 1, and can be summarized as follows.

**STEP 1.** Compute an \( n \times (n-q) \) matrix \( Q \) of full rank that satisfies (25).

**STEP 2.** Set \( U = C^T \), and solve optimization Problem 1 for \( \gamma, \kappa_Y, \kappa_L, Y_Q, Y_C \), and \( L_C \), with

\[ Y = QY_Q Q^T + C^T (CC^T)^{-1} C \]
\[ L = L_C C \quad (31) \]

**STEP 3.** Compute the gain matrix \( K \) using (28) and (30).

**Remark 6.** The design strategy outlined above can be generalized to include matrices \( U \neq C^T \) as well. However, this extension requires the solution of a singular system of equations, and a special construction procedure for matrix \( U \).

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REQUIREMENT 1. Matrix $Y$ must have the form

$$ Y = Y_0 + U_Y^T Y_C U_Y $$

(33)

where $Y_0$ and $Y_C$ are unknown $n \times n$ and $q \times q$ block-diagonal matrices, respectively. The diagonal blocks of $Y_0$ have dimension $n_i \times n_i$, and those of $Y_C$ have dimension $q_i \times q_i$.

REQUIREMENT 2. Matrix $U_Y$ is a user-defined block-diagonal matrix of dimension $n \times n$, consisting of $n_i \times n_i$ blocks.

REQUIREMENT 3. Matrix $Y_0$ must satisfy the equality constraint

$$ Y_0^T C_D Y = U_D $$

(34)

REQUIREMENT 4. Matrix $L$ must have the form

$$ L = L_C U_Y^T $$

(35)

where $L_C$ is a block-diagonal $m \times q$ matrix with blocks of dimension $m_i \times q_i$.

It is easily verified that Requirement 3 implies $LY^{-1} = K_D C_D$, with

$$ K_D = L_C(I - U_Y^T S R) $$

(36)

Requirements 1, 2 and 4 also ensure that $K_D = \text{diag}\{K_{11}, \ldots, K_{NN}\}$ is a block-diagonal matrix, with blocks $K_{ii}$ of dimension $m_i \times q_i$.

3.2 Decentralized Control with a Global Low-Rank Enhancement

Although the decentralized control design described in Section 2 was successfully applied to a number of practical problems (e.g. Šiljak et al., 2002; Zečević et al., 2004), it still faces several significant challenges. The following two are of particular importance:

(i) There are certain classes of systems that are controllable, but cannot be stabilized by decentralized feedback alone. Systems of this type have been studied extensively in the context of decentralized (and structurally) fixed modes (Wang and Davison, 1973; Sezer and Šiljak, 1981; Anderson and Clements, 1981; Davison and Özgüner, 1983).

(ii) Decentralized control designs based on linear matrix inequalities generally require a block diagonal Lyapunov function. Such a constraint is often restrictive, and can significantly degrade the robustness of the closed-loop system. In some cases, it can even lead to infeasibility of the optimization.

With that in mind, our objective in this section will be to supplement decentralized feedback laws with a low-rank centralized correction. We propose to do this by designing a feedback of the form

$$ u_i = K_D x_i + W_V x \quad (i = 1, 2, \ldots, N) $$

(37)

where $W$ and $V$ are matrices of dimension $m_i \times r$ and $r \times n$, respectively. The rank of these matrices is determined by the user, the only constraint being that $r \leq n$.

Defining matrix $W = [W_1^T, \ldots, W_N^T]^T$, the overall control law can be expressed as

$$ u = (K_D + WV)x $$

(38)

where $K_D = \text{diag}\{K_{11}, \ldots, K_{NN}\}$ corresponds to decentralized feedback, and the product $WV$ represents a correction of rank $r$.

To obtain a control of the form (37), we will look for a solution of Problem 1 in the form

$$ Y = Y_D + U_Y^T Y_C $$

$$ L = L_D + L_C U_Y^T $$

(39)

where:

1) $Y_D$ is an unknown symmetric block diagonal matrix, with blocks of dimension $n_i \times n_i$.

2) $L_D$ is an unknown block diagonal matrix, with blocks of dimension $m_i \times n_i$.

3) $U$ is a fixed $n \times r$ matrix.

4) $Y_C$ is an unknown symmetric $r \times r$ matrix.

5) $L_C$ is an unknown matrix of dimension $m \times r$.

For any given choice of $U$, Problem 1 becomes an LMI optimization problem for $Y$, $K_D$, $K_L$, $Y_D$, $Y_C$, $L_C$ and $L_D$. As in the case of static output feedback, the Sherman-Morrison formula allows us to express the gain matrix in a factorized form. In this particular case, we obtain $K = K_D + WV$, where

$$ K_D = L_D Y_D^{-1} $$

(40)

is the decentralized control term and

$$ W = L_C(I - U_Y^T S R) - L_D S R $$

(41)

$$ V = U_Y^T Y_C^{-1} $$

(42)

are matrices of dimension $m \times r$ and $r \times n$, respectively.
The implementation of such a control in a multiprocessor environment is also quite straightforward. Indeed, if matrices \( W \) and \( V \) are partitioned as
\[
W = [W_1^T, \ldots, W_N^T]^T; \quad V = [V_1, \ldots, V_N]
\]
the corresponding control scheme for processor \( i \) has the form shown in Fig. 6. In this scheme, processor \( i \) performs multiplications involving matrices \( W_i \), \( V_i \) and \( K_{ii} \), which are of dimension \( m_i \times r \), \( r \times n_i \) and \( n_i \times n_i \), respectively.

![Fig. 6. Computation tasks for processor \( i \).](image)

This strategy also requires a front-end processor, whose main function is to assemble and distribute the subsystem information, and to form the \( r \times 1 \) vector
\[
z(t) = \sum_{j=1}^{N} V_j x_j(t) = Vx(t)
\]
The only communication tasks involved are single-node gather and scatter operations, which are known to result in low overhead (e.g., Bertsekas and Tsitsiklis, 1989). If necessary, the front-end processor can also periodically recompute matrices \( W_i \), \( V_i \) and \( K_{ii} \), in response to changes in the system configuration.

4. DIMENSIONALITY CONSTRAINTS ON DYNAMIC CONTROLLERS

The third class of constraints on the gain matrix is associated with dynamic controllers, which are required to be of a prescribed order. In order to see how this requirement can be incorporated into the framework of Problem 1, let us consider the nonlinear system (1) with a dynamic controller
\[
\dot{z} = \Pi z + \Gamma y + u_f
\]
of a prescribed order \( r \) (\( r \leq n \)). For moderately sized problems, matrices \( \Pi \) and \( \Gamma \) can be chosen using standard controller reduction techniques, and their suitability is subsequently verified in the course of LMI optimization. In the case of large-scale systems, however, such an approach poses major computational challenges, and alternative methods need to be explored. A possible solution to this problem has been proposed in (Zečević and Šiljak 2004b), based on a combination of balanced realizations and epsilon decomposition. Preliminary dynamic controllers obtained in this manner were found to be very good preconditioners for the design strategy described below.

If we assume a control law of the form
\[
u = Fz + u_s
\]
and introduce vectors \( \xi = [x^T \ z^T]^T \), \( \tilde{u} = [u^T \ z_f^T]^T \)
and \( \tilde{h}(\xi) = [h(x)^T \ 0]^T \), the closed-loop system becomes
\[
\dot{\xi} = \tilde{A}\xi + \tilde{h}(\xi) + \tilde{B}\tilde{u}
\]
with composite matrices
\[
\tilde{A} = \begin{bmatrix} A & BF \\ \Gamma C & \Pi \end{bmatrix}; \quad \tilde{B} = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}
\]
If we also define
\[
\tilde{H} = \begin{bmatrix} H & 0 \\ 0 & 0 \end{bmatrix}
\]
it is readily observed that inequality (2) implies
\[
\tilde{h}^T(\xi)\tilde{h}(\xi) \leq \alpha^2 z^T \tilde{H}^T \tilde{H} \xi
\]
The supplemental inputs \( u_s \) and \( u_f \) are essential for securing system robustness with respect to uncertain nonlinearities. In the following, they will be used to implement a state feedback law
\[
\tilde{u} = \tilde{K}\xi
\]
which guarantees stability for any nonlinearity that satisfies (2).

Matrix \( \tilde{K} \) can be computed by solving Problem 1 with matrices \( \tilde{A}, \tilde{B} \) and \( \tilde{H} \). As noted earlier, if Problem 1 is feasible, the closed-loop system
\[
\dot{\xi} = (\tilde{A} + \tilde{B}\tilde{K})\xi + \tilde{h}(\xi)
\]
is guaranteed to be asymptotically stable for any nonlinearity \( \tilde{h}(\xi) \) that conforms to (50). Partitioning matrix \( K \) in accordance with (48) as
\[ K = \begin{bmatrix} \tilde{K}_{11} & \tilde{K}_{12} \\ \tilde{K}_{21} & \tilde{K}_{22} \end{bmatrix} \quad (53) \]

declares the closed-loop system (52) takes the form
\[
\dot{x} = (A + B\tilde{K}_{11})x + B(F + \tilde{K}_{12})z + h(x) \\
\dot{z} = (\Pi + \tilde{K}_{22})z + (\Gamma C + \tilde{K}_{21})x 
\quad (54) \]

From the standpoint of implementation, such a design has limited practical value, since it assumes that all states are available for control purposes. As a result, it is necessary to additionally require that \( \tilde{K}_{11} \) and \( \tilde{K}_{21} \) have the form
\[
\tilde{K}_{11} = S_{11}C; \quad \tilde{K}_{21} = S_{21}C \quad (55) \]

where \( S_{11} \) and \( S_{21} \) are constant matrices of dimensions \( m \times q \) and \( r \times q \), respectively. If (55) holds, it is easily verified that the resulting closed-loop system
\[
\dot{x} = Ax + BS_{11}y + B(F + \tilde{K}_{12})z + h(x) \\
\dot{z} = (\Pi + \tilde{K}_{22})z + (\Gamma S_{21})y 
\quad (56) \]

becomes a combination of static output feedback and a dynamic controller of order \( r \).

In order to incorporate condition (55) into the LMI optimization, we will consider a solution of Problem 1 in the form
\[
Y = Y_0 + UY_1U^T \\
L = LCU^T 
\quad (57) \]

where
\[
Y_0 = \begin{bmatrix} Y_{11} & 0 \\ 0 & Y_{22} \end{bmatrix} \quad (58) \]

and
\[
Y_{11} = QY_0Q^T + C^T(\Gamma C)^{-1}C \quad (59) \]

The relevant quantities in (57)-(59) are defined as follows:

1) \( Q \) is an \( n \times (n-q) \) matrix of full rank such that \( Q^TQ = 0 \).
2) \( U \) is a fixed \( (n+r) \times r \) matrix of full rank, which can be partitioned as \( U = [U_1^T \ U_2^T]^T \).

Components \( U_1 \) and \( U_2 \) of this matrix are of dimensions \( n \times r \) and \( r \times r \), respectively.

3) Matrix \( U_2 \) can be chosen arbitrarily, but \( U_1 \) is assumed to have the form \( U_1 = C^TM^T \), where \( M \) is a user-defined matrix of dimension \( r \times q \).

4) \( Y_Q, Y_{22}, Y_C \) and \( L_C \) are new LMI variables in Problem 1, replacing \( Y \) and \( L \).

By virtue of the Sherman-Morrison formula, the gain matrix can be factorized as \( K = WV \), where
\[
W = L_C(I - U^TSR) \\
V = U^TY_0^{-1} = [U_1^TY_1^{-1} \ U_2^TY_{22}^{-1}] \quad (60) \]

It is easily verified that the definition of \( Y_1 \) in (59) ensures \( Y_{11}C^T = C^T \), which implies \( U_1^TY_1^{-1} = MC \); matrix \( V \) can therefore be expressed as \( V = [MC \ U_2^TY_{22}^{-1}] \). If we now partition matrix \( W \) in accordance with (58) as \( W = [W_1^T \ W_{21}^T]^T \), it follows that
\[
\hat{K} = \begin{bmatrix} W_{11}MC & W_{11}U_1^TY_{22}^{-1} \\ W_{21}MC & W_{21}U_1^TY_{22}^{-1} \end{bmatrix} \quad (61) \]

which obviously satisfies condition (55).

5. CONCLUSIONS

In this paper we described a number of new results related to the control of large-scale systems under information structure constraints. Our main objective was to show that linear matrix inequalities provide a general design framework that can incorporate both structural and algebraic constraints on the gain matrix. The proposed LMI-based approach was applied to obtain decentralized, overlapping and BBD control laws, as well as static output feedback. In all cases, it was shown that the closed-loop system can tolerate a wide range of nonlinear uncertainties, and that its robustness can be improved by low-order static and dynamic global enhancements.

REFERENCES


