PN Junctions - A. Fabrication

- PN junctions form the basis for most semiconductor devices. Therefore, understanding their operation is basic to the understanding of most devices.
- PN junctions can be fabricated in a variety of techniques:
  - Mask → Implant → Drive-in.
  - The final impurity profile can be simplified as an $erfc$ or Gaussian.

Step Junction - shallow high $N$

Linearly Graded - deep junctions
B. PN Junctions at Equilibrium

Diffusion of carriers from high → low concentration region leaves behind $N_D^+$ and $N_A^-$. 
∴ $\varepsilon$ field is set up which tends to pull $e^-$ and holes back to the original positions.

Equilibrium: $\text{Drift} = \text{Diffusion}$

The general relationship between the charge distribution and the potential is given by Poisson’s equation.

$$\frac{d^2 V}{dx^2} = \frac{q}{K \varepsilon_0} [(n - p) + (N_A - N_D)]$$ (1)

The $e^-$ and hole concentrations are given by:

$$n = n_i e^{\frac{E_F - E_i}{kT}}$$ (2)

$$p = n_i e^{\frac{E_i - E_F}{kT}}$$ (3)
PN Junctions at Equilibrium

Equations (1) – (3) are applicable in the neutral, depletion, and transition regions under equilibrium (i.e. no applied bias etc.).

We will analyze these regions and develop first order theory for the PN junction diodes.
PN Junctions at Equilibrium - Neutral Regions

Neutral regions

N region:
\[ n \cong N_D \]
\[ p \cong n_i^2/N_D \]
\[ n - p - N_D + N_A = 0 \text{ (no net charge)} \]
\[ \therefore \frac{d^2V}{dx^2} = 0, \text{ i.e. the potential is constant in the N region} \]

Thus, from (2):
\[ \frac{E_F - E_i}{q} = \frac{kT}{q} \ell n \frac{N_D}{n_i} \equiv \Phi_n \quad (4) \]

Similarly, in the P neutral region, the potential is obtained from (3):
\[ \frac{E_F - E_i}{q} = -\frac{kT}{q} \ell n \frac{N_A}{n_i} \equiv \Phi_p \quad (5) \]
Band Diagrams at Equilibrium - Built-in Potential

Thus, we have:

\[ \Phi_{bi} \equiv \text{Built-in potential (V}_{bi}\text{)} \]
\[ = \text{potential difference between N-side and P-side neutral regions} \]
\[ = \Phi_n + \Phi_p \]

\[ \therefore \Phi_{bi} = \frac{kT}{q} \left[ \ell_n \frac{N_D}{n_i} + \ell_n \frac{N_A}{n_i} \right] \quad (6) \]

Or,
\[ \Phi_{bi} = \frac{kT}{q} \left[ \ell_n \frac{N_D N_A}{n_i^2} \right] \quad (7) \]
Band Diagrams at Equilibrium - Built-in Potential

- The built-in potential ($\phi_{bi}$) given by (7) exists across a PN junction without an applied bias (thermal equilibrium).
  - Polarity: N$^+$, P$^-$.  
  - This is needed to counteract diffusion.
  - Typical value:
    - 0.5 – 1.0 V in Silicon.
    - 1.0 – 1.4 V GaAs.

- Note:
  - An impurity doping change N$^+$ to N$^-$ or P$^+$ to P$^-$, also, results in a built-in potential.
  - The built-in potential across a PN junction increases as $N_D$ or $N_A$ increases.
Band Diagrams at Equilibrium

**Depletion regions**

If this region is completely depleted of carriers, then:

\[
\frac{d^2V}{dx^2} = \frac{q}{K\varepsilon_o} \left[(N_A - N_D)\right]
\]

Solution of this equation requires some assumption about the spatial variation of \(N_D\) and \(N_A\) as we will see shortly.

**Transition regions**

In this regions between the depletion and neutral regions, neither our depletion nor neutrality approximations are valid and we have to solve (1) – (3).

General solutions across the transition regions require computer techniques. Using such techniques, the transition region is found to be \(\approx 3L_D\), where \(L_D \equiv \) extrinsic Debye length.
PN Junctions - Transition Regions

Debye length is given by:

\[ L_D = \left( \frac{K \varepsilon_o kT}{q^2 |N_D - N_A|} \right)^{1/2} \]  

(9)

- Physically, \( L_D \) may be thought of as the distance over which a fixed charge (\( N_D^+ \) or \( N_A^- \)) exhibits an influence over mobile carrier concentrations.

- It is a measure of the abruptness of the transition regions between the depletion regions and the neutral regions.

1. \( \rho \equiv 0 \) outside depletion region.
2. \( \rho \equiv |N_A - N_D| \) within depletion region.
3. Boundary layer distance \( \approx 3L_D \).
C. Step Junctions

Here, we make some assumptions:

a. \( N_D = \text{constant in N material} \)
   \( N_A = \text{constant in P material} \)

\( \rho = qN_A \) for \( 0 > x > -x_p \)
\( \rho = qN_D \) for \( 0 < x < x_n \)
\( \rho = 0 \) for \( x > x_n; x < -x_p \)

Depletion Approximation
(ignores boundary layers)

\[
\begin{align*}
\vdots \frac{d^2V}{dx^2} &= -\frac{q}{K\varepsilon_o} N_D \quad \text{for } 0 < x < x_n \\
\frac{d^2V}{dx^2} &= \frac{q}{K\varepsilon_o} N_A \quad \text{for } -x_p < x < 0
\end{align*}
\]
Step Junctions - Depletion Approximation

From neutrality, \( N_A x_p = N_D x_n \) and the total depletion layer width is: \( W = x_n + x_p \) (12)

Now, taking the N-side of the junction as an example, we have:

\[
\varepsilon = -\frac{dV}{dx} = -\int_{x_n}^{x} -\frac{q N_D}{K \varepsilon_o} \, dx
\]

Or,

\[
\varepsilon = \frac{q N_D}{K \varepsilon_o} (x - x_n)
\] (13)

Similarly, in the P-side of the junction:

\[
\varepsilon = -\frac{dV}{dx} = -\int_{-x_p}^{x} \frac{q N_A}{K \varepsilon_o} \, dx
\]

Or,

\[
\varepsilon = -\frac{q N_A}{K \varepsilon_o} (x + x_p)
\] (14)
Step Junctions - Depletion Approximation

The potential is obtained by a second integration of (13) and (14) with the results shown below:

\[ \rho = qN_A \text{ for } -x_p < x < 0 \]
\[ \rho = qN_D \text{ for } 0 < x < x_n \]

\[ \varepsilon_{\text{max}} = -\frac{qN_D x_n}{K \varepsilon_o} = -\frac{qN_A x_p}{K \varepsilon_o} \]

\[ \varepsilon \text{ varies linearly between } 0 \text{ and } \varepsilon_{\text{max}} \]

\[ \phi_n = \frac{qN_D x_n^2}{2K \varepsilon_o} + \phi_o \]
\[ \phi_p = \frac{qN_A x_p^2}{2K \varepsilon_o} - \phi_o \]
For a one-sided step junction \((N_A >> N_D)\) or \((N_A << N_D)\) we get:

The depletion region is primarily in the lightly doped side of the junction. Most of the potential changes, also, occurs across the lightly doped side.

\[
\begin{align*}
\rho &= qN_A \quad \text{for } -x_p < x < 0 \\
\rho &= qN_D \quad \text{for } 0 < x < x_n \\
\end{align*}
\]

\[
\varepsilon_{\text{max}} = -\frac{q N_D x_n}{K \varepsilon_o} = -\frac{q N_A x_p}{K \varepsilon_o}
\]

\[
\phi_n = \frac{q N_D x_n^2}{2K \varepsilon_o} + \phi_o
\]

\[
\phi_p = \frac{q N_A x_p^2}{2K \varepsilon_o} - \phi_o
\]
Step Junctions - Depletion Approximation

Since $\varepsilon = -dV/dx$, the total potential across the depletion region is simply the area under $\varepsilon$ field curve.

$$\therefore V = -\int \varepsilon dx$$

That is,

$$\phi_{bi} = \frac{1}{2} \varepsilon_{max} (x_n + x_p) = \frac{1}{2} \varepsilon_{max} W \quad (15)$$

Here, $W = \text{total depletion layer width} = (x_n + x_p)$

Again,

$$\phi_{bi} = \phi_n + \phi_p = \frac{q N_D}{2K \varepsilon_o} x_n^2 + \frac{q N_A}{2K \varepsilon_o} x_p^2$$

And, we know from neutrality: $N_A x_p = N_D x_n$. Therefore, we can solve for $x_n$ and $x_p$ to obtain an expression for $W$. 
Step Junctions - Depletion Approximation

We can show that:

\[ x_n = \left[ \frac{2K \varepsilon_o \phi_{bi} N_A}{q N_D (N_A + N_D)} \right]^{1/2} \]

\[ x_p = \left[ \frac{2K \varepsilon_o \phi_{bi} N_D}{q N_A (N_A + N_D)} \right]^{1/2} \]

From (15) we get:

\[ W = \frac{2\phi_{bi}}{\varepsilon_{\text{max}}} = \frac{2\phi_{bi}}{[qN_Dx_n/K\varepsilon_o]} = \frac{2\phi_{bi}}{[qN_Ax_p/K\varepsilon_o]} \]

Now, using the relationships for \( x_n \) and \( x_p \) we get:

\[ W = \left[ \frac{2K \varepsilon_o \phi_{bi}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2} \]

(16)

Note:

- \( W \) strongly depends on the doping on the lightly doped side.
- \( W \propto 1/\text{SQRT}(\text{doping concentration on the lightly doped side}) \).
D. PN Junction under Reverse Bias

A built-in potential $\phi_i$ provides a barrier to majority carrier current, i.e., $e^-$: $N \rightarrow P$; holes: $P \rightarrow N$

If we bias the junction externally with the N-side $+$ and P-side $-$, the barrier to majority carrier flow is increased.

Results:
- Applied voltage appears entirely across the depletion region.
- $W$ increases, mobile carriers pulled further away from the junction.
- Minority carrier flow is increased but $I$ is small because they are low in number.
- E-field across the depletion region increases.
PN Junction under Reverse Bias: Capacitance

For step junctions:

\[ W = x_p + x_n = \left[ \frac{2K}{q} \varepsilon_o \left( \frac{1}{N_A} + \frac{1}{N_B} \right) (\phi_{bi} - V_A) \right]^{1/2} \]

\[ \varepsilon_{max} = \frac{2(\phi_{bi} - V_A)}{W} = 2 \left( \frac{Pot.\text{AcrossDep.region}}{Width\text{--}of\text{--}Dep.region} \right) \]

Here, \( \phi_{bi} \) is replaced by \( \phi_{bi} - V_A \).

Note that \( V_A \) is negative for a reverse bias PN junction.

The sign convention:

- Reverse bias \( \Rightarrow \phi_{bi}, V_A \) ADD
- Forward bias \( \Rightarrow \phi_{bi}, V_A \) SUBTRACT
The small signal capacitance of the junction is given by:

\[ C = \frac{dQ}{dV} = K\varepsilon_0/W \]  \hspace{1cm} (22)

where, \( W \) = depletion layer width

The above relationship holds for an arbitrary doping profile, i.e, if we can calculate \( W \), we can calculate the small signal capacitance (C).

By small signal capacitance we mean:

- Apply a D.C. potential \( V \) to establish \( W \).
- Measure the capacitance \( C \) by superimposing a small AC signal, i.e. \(|\text{AC signal}| \ll V \) so that \( W \) is not changed.
Example: Consider a step junction

\[ \frac{Q}{A} = qN_D x_n = qN_A x_p \]  \hspace{0.5cm} (23)

\[ \therefore C = \frac{dQ}{dV} = qN_D (dx_n/dV) \]

\[ = qN_A (dx_p/dV) \]  \hspace{0.5cm} (24)

But from (23),

\[ x_p = \left( \frac{N_D}{N_A} \right) x_n \text{ and } W = x_p + x_n \]

\[ \therefore x_n = W - x_p = W - \left( \frac{N_D}{N_A} \right) x_n \Rightarrow x_n \left[ 1 + \frac{N_D}{N_A} \right] = W \]

\[ \therefore x_n = \frac{W}{1 + \frac{N_D}{N_A}} = \frac{W}{1 + \frac{N_D}{N_A}} \left[ \frac{2K \varepsilon_o}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \left( \Phi_{bi} - V_A \right) \right]^{1/2} \]  \hspace{0.5cm} (25)
From (24) and (25) we get:

\[
\frac{d x_n}{d V_A} = \frac{1}{N_D} \left[ \frac{K \varepsilon_o}{2q \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (\Phi_{bi} - V_A)} \right]^{\frac{1}{2}}
\]

Or,

\[
C_j = \left[ \frac{qK \varepsilon_o}{2 \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (\Phi_{bi} - V_A)} \right]^{\frac{1}{2}}
\]

(26)

- In the reverse direction, \(C\) falls as the square root of \(V_A\).
- In the forward direction, \(C \rightarrow \infty\) as \(\Phi_{bi}\) is approached.
- In forward bias (23) becomes invalid because of mobile carriers in the depletion region.
PN Junction under Reverse Bias: Capacitance

Equation (26) can be expressed as:

\[
C_j = \frac{qK \varepsilon_o}{\sqrt{2 \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \Phi_{bi} \left( 1 - \frac{V_A}{\Phi_{bi}} \right)}}
\]

\[
= \frac{C_{j0}}{\sqrt{1 - V_A / \Phi_{bi}}}
\]

(27)

Here at \( V_A = 0 \):

\[
C_{j0} = \frac{qK \varepsilon_o}{\sqrt{2 \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \Phi_{bi}}}
\]

(28)

If \( m \) is the junction grading coefficient:

\[
C_j = \frac{C_{j0}}{(1 - V_A / \Phi_{bi})^m}
\]

(29)
E. Current Voltage Characteristics

To this point we have described the behavior of PN junctions under equilibrium and reverse bias. Under these conditions the currents are small. We now consider FORWARD bias under which substantial currents can flow.

\[ E_i = \phi_p + \phi_n = \text{Built-in potential} \]

Under forward bias, the barrier to majority carrier flow is reduced. And,

- \( e^- \) are injected from N \( \rightarrow \) P.
- \( h^+ \) are injected from P \( \rightarrow \) N.
Minority Carriers

• We would expect that as $\phi_{bi}$ is reduced by $V_A$, the currents which flow should be exponentially dependent on $V_A$. (From FD statistics, # of carriers with $E > q(\phi_{bi} - V_A)$ is exponentially dependent on $V_A$).

• Once $e^-$ go from N $\rightarrow$ P, they become minority carriers. Similarly, for holes. Therefore, minority carrier behavior is of fundamental importance in PN junctions.

• The minority carriers injected across the barrier will tend to recombine if given sufficient time. They will, also, tend to diffuse away from the region of the junction.

• The first thing we need to do is to relate # of injected carriers to $V_A$ using the expression for $\phi_{bi}$:

$$\phi_{bi} = \frac{kT}{q} \left[ \ln \frac{N_D N_A}{n_i^2} \right]$$  \hspace{1cm} (7)
Carrier Density

We define:

\[ n_{po} = \# \text{ e}^- \text{ in P region in thermal equilibrium} \]
\[ p_{no} = \# \text{ holes in N region in thermal equilibrium} \]
\[ n_{no} = \# \text{ e}^- \text{ in N region in thermal equilibrium} \]
\[ p_{po} = \# \text{ holes in P region in thermal equilibrium} \]

Therefore,

\[ n_{no} \cong N_D \text{ and } p_{no} \cong n_i^2/N_D \quad (30) \]
\[ p_{po} \cong N_A \text{ and } n_{po} \cong n_i^2/N_A \quad (31) \]

Using (30) & (31) in (7) we get:

\[ \Phi_{bi} = \frac{kT}{q} \ln \frac{n_{no}}{n_{po}} \quad (32) \]
Carrier Density

We can show that:

\[ n_{n_o} = n_{p_o} e^{\frac{q\phi_{bi}}{kT}} \quad (33) \]

\[ p_{p_o} = p_{n_o} e^{\frac{q\phi_{bi}}{kT}} \quad (34) \]

Thus, the minority carrier concentrations across the space charge regions are related to the majority carrier concentrations on the other side by \( \phi_{bi} \).

Under the applied bias, we replace \( \phi_{bi} \) by \( (\phi_{bi} - V_A) \) in (33) & (34). (We have eliminated subscript (o) to represent non-equilibrium):

\[ n_n = n_p e^{\frac{q(\phi_{bi} - V_A)}{kT}} \quad (35) \]

\[ p_p = p_n e^{\frac{q(\phi_{bi} - V_A)}{kT}} \quad (36) \]
Low Level Injection

We now assume that \( n_n = n_{no} \) and \( p_p = p_{po} \), i.e. the injected carrier densities are small compared to the background concentrations.

\[ n_n = n_{p_o} e^{\frac{q \phi_{bi}}{kT}} \]

\[ n_n = n_{p_o} e^{\frac{q(\phi_{bi} - V_A)}{kT}} \]

From (35):

Then we can show:

\[ n_p = n_{p_o} e^{\frac{qV_A}{kT}} \quad (37) \]

\[ p_n = p_{n_o} e^{\frac{qV_A}{kT}} \quad (38) \]

(37) & (38) define the minority carrier densities at the edge of the space charge region under applied bias. They represent boundary conditions that we will use in calculating current densities.
Low Level Injection

In summary, under low level injection we have:

- The injected carriers in the N and P regions momentarily set up an $\varepsilon$ field.
- This field draw in majority carriers in each region.
- These majority carriers neutralize the injected carriers and re-establish neutrality.
- While this process is going on, the injected minority carriers diffuse into the N and P regions. That is recombination process takes place over some distance.

What happens next?

1. The injected carriers in the N and P regions momentarily set up an $\varepsilon$ field.
2. This field draw in majority carriers in each region.
3. These majority carriers neutralize the injected carriers and re-establish neutrality.
4. While this process is going on, the injected minority carriers diffuse into the N and P regions. That is recombination process takes place over some distance.
Low Level Injection

Majority carrier concentration is essentially unchanged.

The injected hole concentration decays to equilibrium level over some distance.

The \( \varepsilon \) field that exists in the regions of excess carrier concentration is:

\[
N \text{ region: } I_{\text{drift}} = q\mu_n n\varepsilon \quad \text{for majority carrier e}^- \\
I_{\text{drift}} = q\mu_p p\varepsilon \quad \text{for minority carrier holes}
\]

Since \( n \gg p \), the drift field on holes is negligible. The minority carriers move primarily by diffusion while the majority carriers are pulled to the junction by drift (\( \varepsilon \)).
Low Level Injection

Thus, we see that the minority carriers really control the behavior of PN junctions.

We assume that the injected minority carriers move away from the depletion region only by diffusion. This is known as the DIFFUSION APPROXIMATION.

Therefore, in the N-region the minority carrier current:

\[ I_p = -qA D_p \frac{d p_n}{dx} \]  \hspace{1cm} (39)

Substituting in the hole continuity equation we get:

\[ \frac{d}{dt} p_n = D_p \frac{d^2 p_n}{dx^2} - \frac{p_n - p_{n_0}}{\tau_p} \]  \hspace{1cm} (40)
Low Level Injection

Similarly, in the P-region we get:

\[ I_n = qA D_n \frac{d n_p}{d x} \]  \hspace{1cm} (41)

\[ \frac{d n_p}{dt} = D_n \frac{d^2 n_p}{d x^2} - \frac{n_p - n_{p_o}}{\tau_n} \]  \hspace{1cm} (42)

Solving Eqn (39) - (42) with appropriate boundary conditions, we can find the current flowing in a diode with an applied bias \( V_A \).

We will solve (40) and (42) for two specific type of PN junction diodes to derive the expressions for currents.
Long Base Diode

Recombination reduces $p_n$ and $n_p$ to equilibrium values before they reach ohmic contacts.

In the steady state:

\[
\frac{d p_n}{dt} = D_p \frac{d^2 p_n}{dx^2} - \frac{\nabla p_n}{\tau_p} = 0
\]

\[
\frac{d n_p}{dt} = D_n \frac{d^2 n_p}{dx^2} - \frac{\nabla n_p}{\tau_n} = 0
\]
Long Base Diode

The general solutions of these equations are:

\[
\nabla p = K_1 e^{-\frac{x}{\sqrt{D_p \tau_p}}} + K_2 e^{\frac{x}{\sqrt{D_p \tau_p}}} \\

\nabla n = K_3 e^{-\frac{x}{\sqrt{D_n \tau_n}}} + K_4 e^{\frac{x}{\sqrt{D_n \tau_n}}}
\]

(43) (44)

The appropriate boundary conditions are:

\[
p_n = p_n(0) = p_{n_0} e^{qV_A/kT} \quad @ \ x = x_n \]
\[
= p_{n_0} \quad @ \ x = \infty
\]
\[
n_p = n_p(0) = n_{p_0} e^{qV_A/kT} \quad @ \ x = -x_p \]
\[
= n_{p_0} \quad @ \ x = -\infty
\]

Using these boundary conditions in (43) and (44) we can calculate the excess carrier density.
Long Base Diode

Thus, we have:

\[ \nabla p = p_{n_0} \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{-\frac{x-x_n}{L_p}} x + \]  
(45)

\[ \nabla n = n_{p_0} \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{-\frac{x+x_p}{L_n}} x - \]  
(46)

where,

\[ L_p = \sqrt{D_p \tau_p} = \text{holeDiffusionLength} \]

\[ L_n = \sqrt{D_n \tau_n} = e^- \text{DiffusionLength} \]

The hole and \( e^- \) currents are given by substitution into (39) & (41):

\[ I_p = qA \left( \frac{D_p p_{n_0}}{L_p} \right) \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{-\frac{x-x_n}{L_p}} \]  
(47)

\[ I_n = qA \left( \frac{D_n n_{p_0}}{L_n} \right) \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{-\frac{x+x_p}{L_n}} \]  
(48)
Long Base Diode

Note:

1. The minority carriers decrease with distance away from the junction.

3. From continuity of $I$, the majority currents must $\uparrow$ with distance. That is, the minority current is continuously transformed into majority carrier current.

\[ \therefore \text{Total current} = I_p + I_n \text{ at the edges of the depletion region.} \]
Long Base Diode

\[ I_{TOT} = I_p|_{x=x_n} + I_n|_{x=-x_p} \]

From (47) and (48) we get:

\[
I = qA \left( \frac{D_p P_{n_o}}{L_p} + \frac{D_n n_{p_o}}{L_n} \right) \left( \frac{qV_A}{kT} - 1 \right) \tag{49}
\]

Or,

\[
I = I_o \left( \frac{qV_A}{kT} - 1 \right) \tag{50}
\]

Where

\[
I_o = qA \left( \frac{D_p P_{n_o}}{L_p} + \frac{D_n n_{p_o}}{L_n} \right)
\]

Or,

\[
I_o = qA n_i^2 \left( \frac{D_p}{N_D L_p} + \frac{D_n}{N_A L_n} \right) \tag{51}
\]
Long Base Diode

Eqn. (51) illustrates that the doping level on the LIGHTLY DOPED side of the junction determines the current.

Example: If N is lightly doped

\[ p_{no} = \frac{n_i^2}{N_D} \gg n_{p_o} = \frac{n_i^2}{N_A} \]

∴ \( p_n \gg n_p \) and N side controls the amount of current

Therefore, the current is mostly due to the injection into the lightly doped side of PN junction.
A second limiting case of considerable practical importance occurs when the length of the “neutral’ regions are $\ll L_P$ and/or $L_N$. An example is the base region of a bipolar transistor.

- The surface recombination velocity at the ohmic contacts is $\approx \infty$ because of the large number of available traps. Therefore,
  
  \[ p_n(x = W_N) = p_{no} \text{ and } n_p(x = -W_P) = n_{po}. \]

If we assume that no recombination occurs within $W_N$ and $W_P$, then the minority-carrier currents (due to diffusion) are given by:
Short Base Diode

\[ \frac{I_p}{A} = -qD_p \frac{dp}{dx} \quad (52) \]
\[ \frac{I_p}{A} = qD_n \frac{dn}{dx} \quad (53) \]

These currents must be constant (no majority-carrier current flows because no recombination).

\[ \frac{dp}{dx} = \text{constant} \quad \text{and} \quad \frac{dn}{dx} = \text{constant} \]

Therefore, hole and e\(^{-}\) concentrations decrease linearly with distance as shown (in Fig. on slide #46).

Under these conditions:

\[ \frac{I_p}{A} = -q D_p \frac{dp}{dx} = -q D_p \frac{\nabla p}{W_n} \]

\[ = -q D_p \frac{p_{n_0}}{W_n} \left( e^{\frac{qV_A}{kT}} - 1 \right) \]

\[ = -q D_p \frac{n_i^2}{N_D W_n} \left( e^{\frac{qV_A}{kT}} - 1 \right) \quad (54) \]
Short Base Diode

Similarly,

\[
I_n = q D_n \frac{n_i^2}{N_A W_P} \left( \frac{qV_A}{e kT} - 1 \right) \quad (55)
\]

Then the total current is given by the sum of the two minority currents and is given by:

\[
I = qA n_i^2 \left[ \frac{D_p}{N_D W_N} + \frac{D_n}{N_A W_P} \right] \left( \frac{qV_A}{e kT} - 1 \right)
\]

Or,

\[
I = I_o \left( \frac{qV_A}{e kT} - 1 \right) \quad (56)
\]

Comparing (56) and (50) (long base diode) we see that the two equations are identical except for the characteristic length appropriate to the geometry (\(L_{p,n}\) or \(W_{P,N}\)).
**I - V Characteristics - Summary**

In summary we have:

\[
I_o = \begin{cases} 
q A n_i^2 \left( \frac{D_p}{N_D L_p} + \frac{D_n}{N_A L_n} \right) & \Leftrightarrow L.B \\
q A n_i^2 \left( \frac{D_p}{N_D W_N} + \frac{D_n}{N_A W_P} \right) & \Leftrightarrow S.B 
\end{cases}
\]

\[
I = I_o \left( \frac{qV_A}{e^{kT}} - 1 \right)
\]

Actual diodes may represent intermediate cases. In either case it is the lightly doped side of the junction which largely determines I.
Assumptions:

1. **Ohmic drops** in bulk regions are negligible. Typically, this is true for low level injection.

2. **Quasi-equilibrium** implies injected currents are small compared to normal diffusion and drift currents across the depletion region. This was needed in our calculation of $pn$ and $np$ and is, also, generally valid except high level injection.
F. Space Charge Recombination Currents

We have assumed to this point that all the minority-carriers cross the depletion region. In practice, some recombine through trapping centers. Under reverse bias, these centers act as generation sites, increasing $I_o$ over the ideal value calculated earlier. We know:

$$U = + \Rightarrow \text{recombination under forward bias } (pn > n_i^2)$$

$$U = - \Rightarrow \text{generation under reverse bias } (pn < n_i^2)$$

$$U = \frac{\nu_i \sigma_n N_t (pn - n_i^2)}{n + p + 2n_i \cosh \left[ \frac{E_i - E_t}{kT} \right]}$$
Space Charge Recombination Currents

We have discussed that $U$ is a maximum when $E_t = E_i$.

$$U_{\text{max}} = \frac{\nu_{th} \sigma N_i (pn - n_i^2)}{n + p + 2n_i}$$

Throughout the space charge region under the non-equilibrium condition of injection, we can show:

$$pn = n_i^2 e^{\frac{qV_A}{kT}}$$

$$\nu_{th} \sigma N_i n_i^2 \left( e^{\frac{qV_A}{kT}} - 1 \right)$$

$$U_{\text{max}} = \frac{\nu_{th} \sigma N_i n_i^2 \left( e^{\frac{qV_A}{kT}} - 1 \right)}{n + p + 2n_i}$$

In the space charge region, we can show that $U$ is maximum when $n = p$. ( $n$ & $p$ are both $\approx 0$ for a completely depleted region.)
Space Charge Recombination Currents

The total current (within the depletion region) corresponding to this recombination rate is simply:

\[ I_{\text{rec}} = qA \int_{-x_p}^{x_n} U_{\text{max}} dx \]

\[ \therefore I_{\text{rec}} = \frac{qA n_i W}{2\tau_o} e^{\frac{qV_A}{2kT}} \]

where \( W \) is the total width of the depletion region.

Note:
- Under forward bias, \( I_{\text{rec}} \) will dominate at small \( V_A \) but will be swamped out by the ideal diode current at high \( I \).
- Under reverse bias, \( I_{\text{rec}} \) typically dominates \( I_o \) which is a constant for the ideal diode. Since in the depletion region \( pn << n_i^2 \).

\[ \therefore U_{\text{max}} \cong - \frac{n_i}{2\tau_o} \Rightarrow I_{\text{rec}} = \frac{qA n_i W}{2\tau_o} \]
G. Charge Storage in PN Diodes

Consider the long base diode shown below (one sided):

Since \(|P+| \gg |N|\)

essentially

no injection

in P+ region.

If the forward bias is removed at \(t = 0\) and replaced by a reverse bias, the stored holes will flow back across the depletion region resulting in a reverse current until they are all removed.

The driving force \((V_F)\) for injected holes is removed at \(t = 0\). The concentration decreases to equilibrium by backwards flow across the barriers.
Charge Storage in PN Diodes

The total charge stored in the N side of the diode is given by:

\[ Q_s = qA \int_{0}^{W_N} \Delta p dx \]  

(58)

The continuity equation for holes is given by:

\[ \frac{1}{q} \frac{\partial J_p}{\partial x} + \frac{\Delta p}{\tau_p} = - \frac{\partial p}{\partial t} \]  

(59)

Integrating from \( x = 0 \) to \( x = W_N \):

\[ \frac{1}{q} \int_{0}^{W_N} \frac{\partial J_p}{\partial x} dx + \int_{0}^{W_N} \frac{\Delta p}{\tau_p} dx = - \int_{0}^{W_N} \frac{\partial p}{\partial t} dx \]

Using (58) we get:

\[ \frac{1}{q} \left[ J_p(W_N) - J_p(0) \right] + \frac{Q_s}{qA\tau_p} = - \frac{1}{qA} \frac{d Q_s}{dt} \]
Charge Storage in PN Diodes

\[ I_p(0) - I_p(W_N) = \frac{dQ_s}{dt} + \frac{Q_s}{\tau_p} \quad (60) \]

This is the charge control equation and relates current to stored charge. In the long base diode, \( I_p(W_N) = 0 \) and under steady state, forward biased conditions, \( dQ_s/dt = 0 \). Therefore,

\[ I_F = I_p(0) = Q_s/\tau_p \quad (61) \]

This stored charge is directly proportional to \( I_F \) and \( \tau_p \). If we reverse the current direction, eqn. (60) becomes:

\[ -I_R = dQ_s/dt + Q_s/\tau_p \quad (62) \]

This yields a solution:

\[ Q_s = \tau_p[-I_R + (I_F + I_R)e^{-t/\tau_p}] \quad (63) \]

The stored charge goes to 0 when \( Q_s(t) = 0 \), so that:

\[ t_s = \tau_p \ln(1 + I_F/I_R) \quad (64) \]
Charge Storage in PN Diodes

Note:
- The storage time, $t_s \propto \tau_p$.
- Diode supports full reverse voltage after $t_s$ and an RC decay due to diode capacitance and circuit-diode resistance.
- $t_s$ decreases with increasing $I_R$.
- Higher forward bias $\Rightarrow$ longer $t_s$ because of more storage charge.
H. Circuit Models

A simple PN junction circuit (small-signal) model:

\[ R_s = \text{diode series resistance} \]

\[ g_D = \text{diode conductance (1/resistance)} \]
\[ = \frac{dI}{dV} = \frac{(q/kT)I_0 e^{qV_A/kT}}{qVA/kT} = \frac{(q/kT)I}{qVA/kT} \]

\[ C/A = \text{depletion capacitance} \]
\[ = \sqrt{\frac{qK e_o}{2 \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \phi_{bi} - V_A}} \]  
  (step junction)

\[ C_D/A = \text{diffusion capacitance (due to the variation of minority-carrier stored charge in quasi-neutral regions)} \]
\[ = \frac{(qI_0 \tau_p/kT)e^{qV_A/kT}}{qVA/kT} \]

All of the above diode parameters except \( R_s \) depend on \( V_A \). For large signal calculations, piecewise linear models can be used for computer calculations.