Semiconductor Materials in Equilibrium

Solid materials may be classified as:

1. *Amorphous* - no well defined structure

2. *Polycrystalline* - many small regions each having well organized atomic structure

3. *Crystalline* - long range 3D order of atoms; repeating “unit cell”

Most semiconductor materials fall into the crystalline category.

Three cubic-crystal unit cells: (a) Simple cubic, (b) Body-centered cubic, and (c) Face-centered cubic.
Electric properties of a material depend on:
- fixed positions of atoms within the lattice
- bonding of atoms to their neighbors. Particularly important are the outermost or valence electrons.

In our discussion we will consider Si as a specific example.

- Silicon forms a crystal with the diamond lattice structure.
- The structure is two FCC unit cells offset by (1/4)a.
Silicon Atomic Structure

We know from Quantum Mechanics that each electron must have a separate distinct energy state defined by four quantum numbers:

- Principal quantum number
  \( n = 1, 2, 3, \ldots \)

- Angular momentum quantum number
  \( l = 0, 1, 2, \ldots, n-1 \)

- Magnetic quantum number
  \( m = 0, \pm 1, \ldots \)

- Electron spin
  \( s = \pm 1/2 \)
Silicon Atomic Structure

Electron shells and sub-shells:

<table>
<thead>
<tr>
<th>Shell ( n )</th>
<th>( K ) 1</th>
<th>( L ) 2</th>
<th>( M ) 3</th>
<th>( N ) 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l ) Sub-shell</td>
<td>0 ( s )</td>
<td>0 ( s )</td>
<td>0 ( s )</td>
<td>0 ( s )</td>
</tr>
<tr>
<td></td>
<td>1 ( p )</td>
<td>1 ( p )</td>
<td>1 ( p )</td>
<td>1 ( p )</td>
</tr>
<tr>
<td>Number of electrons</td>
<td>2 6 10</td>
<td>2 6 10</td>
<td>2 6 10</td>
<td>2 6 10</td>
</tr>
</tbody>
</table>

Si has \( K \) and \( L \) shells filled, \( M \) shell is half filled with four valence \( e^- \) and four empty states in the outermost sub shell.

Electronic configuration for Si:

\[ 1s^22s^22p^63s^23p^2 \]
Bonding Model

- Crystalline Silicon:
  - Covalent bonds are formed with 4 nearest neighbors.
  - Valence electrons:
    - are shared in covalent bonds
    - are loosely bound to atoms
    - can become free, especially, at elevated temperatures to contribute to conduction.
Bonding Model

- Thermal energy:
  - breaks bonds
  - liberates electrons
  - creates “holes” or vacancies

- Both $e^-$ and holes are mobile and can contribute to the conduction in the semiconductor.

- Note that $n = p$ in the intrinsic semiconductor.
Energy Band Model

• The fact that e\(^{-}\) can exist in a solid only in distinct energy levels results in certain bands of allowed energy states.

• Valence *electrons* exist in the valence band.

• Conduction *electrons* exist in the conduction band.

Metal

 semiconductor

 Insulator

\[
E_g \approx 1 \text{ eV}
\]

\[
E_g > 5 \text{ eV}
\]
Intrinsic Carriers

• In a semiconductor, as $T$ increases, the atoms have more thermal energy. As a result, the probability of broken bonds increases and $n$ increases.

• Note:
  - $T \uparrow \Rightarrow n_i \uparrow$
  - $E_g \uparrow \Rightarrow n_i \downarrow$

where, $n_i \equiv$ intrinsic carrier concentration.
Extrinsic Concentrations - Dopants

Donor

Acceptor

Conduction electron

Hole

Donor

Acceptor

Conduction electron

Hole
Dopants

• To increase and control the conductivity of semiconductors we add controlled amounts of “dopants” such as:
  – N type: P, As, and Sb each has 5 valence electrons
  – P type: B, Al, and Ga each has 3 valence electrons.

• These atoms occupy substitutional lattice sites and the extra $e^-$ or hole is very loosely bound, i.e. can easily move to the conduction or valence bands, respectively. In terms of band diagrams this can be represented as:

\[ E_c \quad E_v \quad 1.1\text{eV} \quad \approx 0.05\text{eV} \quad E_D \quad \text{Donor} \]

\[ E_c \quad E_v \quad 1.1\text{eV} \quad \approx 0.05\text{eV} \quad E_A \quad \text{Acceptor} \]
Dopants - Donor and Acceptor Levels

Measured *donor* and *acceptor* levels for various impurities in silicon

- *Acceptor* levels are below the gap center and measured from the top of the valance band.
- *Donor* levels are above the gap center and measured from the bottom of the conduction band band.

<table>
<thead>
<tr>
<th>Impurity</th>
<th>Donor Level</th>
<th>Acceptor Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sb</td>
<td>0.039</td>
<td>0.054</td>
</tr>
<tr>
<td>P</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>As</td>
<td>0.045</td>
<td>0.072</td>
</tr>
<tr>
<td>B</td>
<td>0.067</td>
<td>0.16</td>
</tr>
<tr>
<td>Al</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>Ga</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>In</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

Si

\[ 1.12 \]

\[ E_v \]

\[ E_c \]
Equilibrium Carrier Concentrations in Semiconductors

• To predict, quantitatively, electrical behavior of a semiconductor, we need to know:
  – density of states function
  – probability of occupancy of those states.

• When an $e^-$ in the conduction band gains energy, it moves up to an $E > E_c$.

• Similarly, when a hole in the valence band gains energy, it moves down to an $E < E_v$. 

![Diagram of energy bands and carrier movement](image)
Density of States

From quantum mechanical considerations it may be shown that:

\[ g_c(E) = \left(\frac{4\pi}{h^3}\right)\left(2m_e\right)^{3/2}(E - E_c)^{1/2} \]  
(1)

= density of states in the conduction band

\[ g_v(E) = \left(\frac{4\pi}{h^3}\right)\left(2m_h\right)^{3/2}(E_v - E)^{1/2} \]  
(2)

= density of states in the valence band

where,

\[ h = \text{Planck’s constant} = 6.63 \times 10^{-34} \text{ Joules.sec} \]

\[ m_e = \text{effective mass of } e^- \]

\[ m_h = \text{effective mass of hole} \]

(Effective mass of \( e^- \) is different from free \( e^- \) mass because of the solid lattice in which it moves.)
The Fermi Function

- The probability that an allowed state is occupied is described by the Fermi-Dirac distribution function:

\[ f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \]  

- \( E_F = \text{Fermi Level} \equiv \text{energy at which the probability of occupancy} = 1/2 \)
- \( k = \text{Boltzmann constant; } T = \text{temperature.} \)

- Again, the probability of a state not being occupied is:

\[ 1 - f(E) = \frac{1}{1 + e^{(E_F - E)/kT}} \]  

= Probability a hole exists.
The Fermi Function

At $T = 0^\circ\text{K}$
- all states $< E_F$ are occupied.
- all states $> E_F$ are empty.

At $T > 0^\circ\text{K}$
- some $\text{e}^-$ have $E > E_F$.
- some holes exist $E < E_F$. 
The Boltzmann Approximation

For energies $> 3kT$ ($\cong 75$ meV @ room temp) above $E_F$, $f(E)$ can be approximated as:

$$f(E) \approx e^{-\frac{(E - E_F)}{kT}}$$  \hspace{1cm} (5)

Equation (5) represents Maxwell-Boltzmann function.

This approximation is quite good for many device applications, as we will see later.
Equilibrium Carrier Distribution

To find the number of electrons in the conduction band or the number of holes in the valence band, we simply take the product of $f(E)$ and $g(E)$ as shown on the plots.
Equilibrium Carrier Distribution

Mathematically,

\[ n = \int_{E_c}^{\infty} f(E) g(E) \, dE \]  \hspace{1cm} (6)

\[ p = \int_{-\infty}^{E_v} [1 - f(E)] g(E) \, dE \]  \hspace{1cm} (7)

\[ \Rightarrow n = N_c \exp[-(E_c - E_F)/kT] \]  \hspace{1cm} (8)

\[ p = N_v \exp[-(E_F - E_v)/kT] \]  \hspace{1cm} (9)

where,

\[ N_c = 2(2\pi m_e kT/h^2)^{3/2} \]

\[ N_v = 2(2\pi m_h kT/h^2)^{3/2} \]

Eqns (8) & (9) use Maxwell-Boltzmann approximation for \( f(E) \).
Equilibrium Carrier Distribution

N_c \equiv \text{effective density of conduction band states}
\approx 2.8 \times 10^{19} \text{ cm}^{-3} @ 25^\circ \text{C}

N_v \equiv \text{effective density of valence band states}
\approx 1 \times 10^{19} \text{ cm}^{-3} @ 25^\circ \text{C}

m_e = \text{density of states effective mass for electrons in conduction band} \approx 1.18 m_o; (m_o = \text{mass of electron in free space})

m_h = \text{density of states effective mass for holes in valence band} \approx 0.81 m_o

The product of the number of e- and holes is given by:

np = N_c N_v e^{-(E_c - E_v)/kT} = CT^3 e^{-E_g/kT} = n_i^2 \quad (10)

or,

n_i = 3.9 \times 10^{16} T^{3/2} e^{-E_g(T)/2kT}
Equilibrium Carrier Distribution

• At any $T$, $n_p$ is:
  – a constant
  – a function of $f(E)$
  – a function of $g(E)$
  – independent of doping
  – independent of $E_F$.

• $n_p \approx 2.25 \times 10^{20} \text{ cm}^{-6}$ for Si @ $300^\circ$K.
In a pure semiconductor (intrinsic/undoped),
\[ n = p = n_i = \sqrt{(N_c N_v)} e^{-\frac{E_g}{2kT}} \] \hspace{1cm} (11)

From (8) and (9), we can show that:
\[ N_c \exp[-(E_c - E_F)/kT] = N_v \exp[-(E_F - E_v)/kT] \]
\[ \therefore \ E_F = \frac{1}{2}(E_c + E_v) + \frac{3}{4}[kT \ln(m_h/m_e)] \equiv E_i \] \hspace{1cm} (12)
\[ \equiv \frac{1}{2}(E_c + E_v) \] \hspace{1cm} (13)

Combining the above equations, we get:
\[ n = n_i e^{(E_F - E_i)/kT} = n_i e^{q(\phi - \phi_F)/kT} \] \hspace{1cm} (14)
\[ p = n_i e^{-(E_i - E_F)/kT} = n_i e^{q(\phi_F - \phi)/kT} \] \hspace{1cm} (15)

where \( \phi = -E_i/q \) and \( \phi_F = -E_F/q \). These equations are valid for doped as well as undoped materials.
Doped Semiconductors

If we purposely add donors such as P, As, Sb to a semiconductor like Si. Then assuming complete ionization of donor atoms:

\[ n = p + N_D \]  \hspace{1cm} (16)

More generally, charge neutrality requires:

\[ n + N_A = p + N_D \]  \hspace{1cm} (17)

Since \( np = n_i^2 \), substituting for \( p = n_i^2/n \) in (17) we get:

\[ n_n = (1/2)[\sqrt{(N_D^2 + 4n_i^2)} + N_D] \equiv N_D \]  \hspace{1cm} (18)

\[ p_n = (1/2)[\sqrt{(N_D^2 + 4n_i^2)} - N_D] \equiv n_i^2/N_D \]  \hspace{1cm} (19)

But we know that \( n = N_c \exp[-(E_c - E_F)/kT] \)

Therefore, for N-type semiconductors,

\[ E_F = E_c - kT \ln(N_c/N_D) \]  \hspace{1cm} (20)
Temperature and Doping Dependence of $E_F$

We can show that:

\[ E_F - E_i = kT \ln \left( \frac{n}{n_i} \right) \quad \text{or} \quad E_c - E_F \approx kT \ln \left( \frac{N_c}{n} \right) \]

\[ E_i - E_F = kT \ln \left( \frac{p}{n_i} \right) \quad \text{or} \quad E_F - E_v \approx kT \ln \left( \frac{N_v}{p} \right) \]

Note: The expressions are derived using MB statistics, and are not accurate when $E_F \to E_D$, $E_A$ or the band edges.
Temperature Dependence of $E_g$

Assume shallow *donor* and *acceptor* levels such as:

\[
E_c - E_D = 0.044 \text{ eV} \\
E_A - E_v = 0.044 \text{ eV}
\]

As $T$ increases, $n_i$ increases. And, the material becomes more intrinsic. Therefore, $E_F$ approaches $E_i$. Also, $E_g$ decreases as $T$ increases.

\[
E_g (T) = 1.160 - \frac{7.02 \times 10^{-4} T^2}{T + 1108}
\]  

\[
\therefore E_g (300^\circ K) \approx 1.115 \text{ eV}
\]

*Band-gap narrowing* is important in some devices like bipolar transistors which we will deal with later. $E_g$ is much reduced at process temperatures.
Temperature-dependence of Carrier Concentration

Normal operation of semiconductor devices is in the *extrinsic range* of conductivity where we purposely add impurities to control the electrical properties.

At low temperatures, impurity freeze-out occurs due to insufficient thermal energy to excite $e^- \approx 0.05$ eV from $E_D$ to $E_C$. 

$$\frac{n_n}{N_d} \approx 1 \quad \text{for} \quad T \ll T_F$$

$T_F = \frac{k_B N_d}{\mu_0 h}$
Free carriers in Semiconductors

Electronic devices depend on the motion of free carriers in semiconductor crystals. From carrier statistics, we know how to calculate the number of free carriers; Now, let us concentrate on the methods of moving these carriers from one region of a semiconductor to another. We will discuss three different mechanisms:

A) **Thermal** motion

B) **Drift** or Electric field driven motion

C) **Diffusion** or Gradient driven motion.
A. Thermal Motion

In thermal equilibrium, mobile (conduction band) $e^-$ will be in random thermal motion. From statistical mechanics:

Thermal energy $E = (3/2)kT$

Again, the kinetic energy of these $e^-$ = $(1/2)m_e v_{th}^2$

Therefore, $(1/2)m_e v_{th}^2 = (3/2)kT$  

(22)

where, $m_e$ = effective mass of conduction band electrons.

∴ $v_{th} \cong 10^7$ cm/sec @ 300°K (= avg. velocity of thermal motion).

In thermal equilibrium, the thermal motion is random. As a result, the net current is zero.
There are several possible scattering mechanisms for electrons:

1. **Phonon scattering**. Lattice atoms vibrate and from Quantum Mechanics we know that discrete allowed vibrational states exist. This can be modeled as discrete particles called *phonons*. Transfer of energy from electrons to the lattice is called phonon scattering.

2. **Ionized impurity atom scattering** - important at high dopant concentrations.

3. **Neutral impurity atom scattering** - usually negligible.

4. **carrier-carrier (e\(^-\) - e\(^-\) & e\(^-\) - hole) scattering** - important at high carrier concentrations.
5. *Surface scattering effects* - important in MOSFET devices.

6. *Crystal defects* - can be minimized by good crystals.

Mean free path ($\lambda_m$) ≡ average distance between collisions

Typically, for Si:

$$\lambda_m \approx 10^{-6} - 10^{-4} \text{ cm.}$$

Since $v_{th} \approx 10^7 \text{ cm/sec}$, the mean free time between collisions is:

$$\tau_m \approx (10^{-5} \text{ cm})/(10^7 \text{ cm/sec}) \approx 1 \text{ psec.}$$
B. Drift - ε Field Driven Motion

Now, let us apply an ε field across the crystal

\[ \text{Si Crystal} \]

- Motion of $e^-$ is now in the direction opposite to ε.
- This process is called drift and causes a net current flow.

The average $e^-$ velocity called the drift velocity and:

\[ v_d \equiv \mu \varepsilon \quad \text{(23)} \]

where $\mu$ = mobility
Drift - $\varepsilon$ Field Driven Motion

- $v_d = \mu \varepsilon$ linear relationship holds only for small fields.
- Mobility depends on parameters like:
  - material
  - doping
  - temperature
  - scattering mechanisms.
Drift - $\varepsilon$ Field Driven Motion

- At high fields, $v_d$ becomes comparable to $v_{th}$ which is an approximate upper bound of $v_d$. Thus, $v_d$ saturates.

  $v_{sat} \equiv$ scattering limited velocity

  $\approx 1 \times 10^7$ cm/sec for $e^-$ in Si

  $\approx 8 \times 10^6$ cm/sec for holes in Si.

- The general expression for $v_d$ is: $v_d = \frac{\mu_0 \varepsilon}{1 + \left(\frac{\mu_0 \varepsilon}{v_{sat}}\right)^\beta} \frac{1}{\beta}$ (24)

- The critical field at which $v_{sat}$ occurs is: $\approx 2 \times 10^4$ V/cm = 2 V/\(\mu\)m.

  *This is an important physical limit in very small semiconductor devices like MOSFETs.*
µ Vs. Doping Concentration at T = 300 K
$\mu_n$ Vs. Temperature

- Ionized impurity scattering is more important at low $T$.
- In general:

$$\mu = \frac{1}{\mu_L + \mu_I + \ldots}$$

Assume that the different scattering mechanisms operate independently.
Drift - current

The drift of carriers under an applied $\varepsilon$ results in a current flow:

$$I = -qAnv_d$$  \hspace{1cm} (25)

where $A =$ Area

$n =$ number of $e^- \text{ per unit volume}$

Substituting for $v_d$ and considering both $e^-$ and holes

$$\therefore \frac{I}{A} = (nq\mu_n + pq\mu_p)\varepsilon$$ \hspace{1cm} (26)

Usually, the majority carrier-term in (26) is important.

Since $\varepsilon = V/L$, then

$$\frac{(V/L)/(I/A)}{1/(q\mu_n n + q\mu_p p)}$$

where $V =$ applied bias

$L =$ length of semiconductor
Drift - $\varepsilon$ Field Driven Motion

\[
\frac{(V/L)}{(I/A)} = \frac{(V/I)}{(A/L)} = \frac{1}{(q\mu_n n + q\mu_p p)} \quad (27)
\]

We know that:

\[
R = \rho \frac{L}{A} = \frac{V}{I}
\]

or,

\[
\rho = \frac{(V/I)}{(A/L)}
\]

\[
\Rightarrow \quad \rho = \frac{1}{(q\mu_n n + q\mu_p p)} = \frac{1}{\sigma} \quad (28)
\]

where

\[
\rho = \text{resistivity in ohm.cm} \\
\sigma = \text{conductivity in (ohm.cm)}^{-1}
\]

The resistivity of a semiconductor can be measured using

(1) a four-point probe, or (2) Hall effect.
Resistivity Vs. Dopant Density @ T = 300K
C. Diffusion - Gradient Driven Motion

Diffusion occurs from high concentration regions towards low concentration regions.

The diffusion flux obeys Fick’s first law:

\[ F = -D \frac{dn}{dx} \]  \hspace{1cm} (29)

where

- \( F \) = flux of carriers
- \( D \) = diffusion constant
- \( n \) = carrier density.
Transport Equations

The diffusion current through a cross-sectional area is:

\[ I = qAD_n(dn/dx) - qAD_p(dp/dx) \]  \hspace{1cm} (30)

where \( D_n = e^- \) diffusion constant = 38 cm\(^2\)/sec in Si
\( D_p = \) hole diffusion constant = 13 cm\(^2\)/sec in Si

\( D_n \) and \( D_p \) are related to their respective carrier mobilities by the Einstein relationship:

\[ D_p/\mu_p = D_n/\mu_n = kT/q \]  \hspace{1cm} (31)

If both drift and diffusion are important,

\[ I_n = qA[\mu_n n\varepsilon + D_n(dn/dx)] \]  \hspace{1cm} (32)
\[ I_p = qA[\mu_p p\varepsilon - D_p(dp/dx)] \]  \hspace{1cm} (33)

*We will use these basic equations in deriving device characteristics.*
Thermal equilibrium is defined by the condition \( np = n_i^2 \). In thermal equilibrium, the equations discussed earlier may be used to calculate \( E_F, n, p \) etc.

However, the equilibrium condition may be disturbed by the introduction or removal of free carriers.

- **Injection**: Extra carriers are generated by *optical* or *electrical* means, and \( np > n_i^2 \).

- **Extraction**: Here, carriers are removed *electrically*. And, as a result in \( np < n_i^2 \).
Non-equilibrium Semiconductors

• Many important semiconductor devices depend for their operation on non-equilibrium conditions.

• The excess carriers at a given time will recombine to re-establish thermal equilibrium.

• The important parameters that we will introduce in this set of notes are:
  \[ \tau \equiv \text{carrier lifetime (1 nsec - 1 msec)} \]
  \[ R \equiv \text{recombination rate (carriers cm}^{-3} \text{ sec}^{-1}) \]
  \[ G \equiv \text{generation rate (carriers cm}^{-3} \text{ sec}^{-1}) \].

• The mechanism of re-establishing thermal equilibrium is of fundamental importance to the operation of electronic devices.
Generation and Recombination

• At $T > 0^\circ K$ in thermal equilibrium:
  
  – Some valence $e^-$ can gain sufficient energy to jump into the conduction band. This leaves behind a hole, and an $\{e^-,\text{hole}\}$ pair is created (generation $G_{\text{th}}$). Here,

  $$n = p = n_i^2$$  \hspace{0.5cm} \text{(INTRINSIC)}

  – When an $e^-$ makes a transition from the conduction band to the valence band an $\{e^-,\text{hole}\}$ pair is annihilated (recombination $R_{\text{th}}$).  

\[ E_c \quad \Downarrow G_{\text{th}} \quad \Downarrow R_{\text{th}} \quad \Downarrow E_g \]

\[ E_v \quad \Downarrow \quad \Downarrow \]

$E_c$ = Conduction Band

$E_v$ = Valence Band

$G_{\text{th}}$ = Generation

$R_{\text{th}}$ = Recombination

\[ E_g \] = Band Gap
Injection of Carriers

Now, if we shine light on the semiconductor with $E = h\nu > E_g$ so that $e^-$ can be excited into the conduction band.

\[
\begin{align*}
n &= n_i + n_L \\
p &= n_i + p_L \\
n &\geq n_i \\
p &\geq n_i
\end{align*}
\]

\[
\{ \text{INJECTION} \}
\]

\[ h\nu = \text{frequency} \]
\[ h = \text{Planck's constant} \]
Injection of Carriers

Now, if we shine light on an extrinsic semiconductor with:
\[ N_D = 10^{15} \text{ cm}^{-3}. \]

In this case:
\[ n \cong N_D + n_L \]
\[ p \cong n_i^2/N_D + p_L \]
Low level: \( n_L \ll N_D \)
High level: \( n_L \geq N_D \)

In Fig., a low level injection of excess carrier = \( 10^{13} \) is shown.

Since the mathematics for high level injection are complex, we will consider only low level injection.
A. Direct (Band to Band) Recombination

R = rate of recombination

R \propto n \text{ (number of } e^- \text{ in the CB)}

R \propto p \text{ (number of holes in the VB)}

\therefore R = Kpn, \ (K = \text{constant})

Under thermal equilibrium,

\[ G_{th} = R_{th} = Kp_o n_o \]  \hspace{1cm} (34)

where \( p_o \) and \( n_o \) represent the respective equilibrium values.

Suppose, we generate excess carriers (\( \Delta n, \Delta p \)) at a rate \( G_L \) due to incident light.

Then

\[ R = Kpn = K(p_o + \Delta p)(n_o + \Delta n) \]  \hspace{1cm} (35)

where

\[ \Delta p = p - p_o, \]
\[ \Delta n = n - n_o. \]
Direct Recombination

Total generation rate $G = G_{th} + G_L$  \hspace{1cm} (36)

Under steady state (*not thermal equilibrium!!*) condition,

$$G_{th} + G_L = R$$

$$\Rightarrow G_L = R - G_{th} \equiv U = \text{net recombination rate} \hspace{1cm} (37)$$

Substituting (34) and (35) into (37) we get:

$$U = K(p_o + \Delta p)(n_o + \Delta n) - Kp_on_o \hspace{1cm} (38)$$

From charge neutrality, $\Delta p = \Delta n$.

$$\therefore U = K(\Delta p + n_o + p_o)\Delta p \hspace{1cm} (39)$$

For low level injection, $\Delta p \ll (n_o + p_o)$

$$\therefore \Delta p = \frac{U}{K(n_o + p_o)} = U\tau = G_L\tau$$
Direct Recombination

Where $\tau \equiv \frac{1}{K(n_o + p_o)} = \text{excess carrier lifetime (sec)}$.  
$\quad = \text{avg. time excess carrier remains free}$

Thus,  
$$\tau = \frac{\Delta p}{G_L} = \frac{\Delta p}{U} = \frac{\Delta n}{U} = \frac{1}{K(n_o + p_o)} \quad (40)$$

*For band to band recombination, $\tau_n = \tau_p$ as the single phenomena annihilates $e^-$ and hole simultaneously.*

**Example:**
Recombination rate $U = 10^{18}$ carriers cm$^{-3}$ sec$^{-1}$
Minority-carrier lifetime $\tau = 100$ $\mu$sec,

Then from (40), excess-density $\Delta p = \Delta n = (10^{18}) (100 \times 10^{-6}) = 10^{14}$ cm$^{-3}$.

**Summary:**
For a semiconductor with equilibrium carrier concentrations ($n_o$, $p_o$) the steady state carrier concentrations under light are:

$$\begin{align*}
    n &= n_o + \tau G_L \\
    p &= p_o + \tau G_L
\end{align*}$$

$$\begin{align*}
    n_o p_o &= n_i^2 \\
    np &> n_i^2
\end{align*} \quad (41)$$
B. Indirect Recombination (Through Trapping Levels)

In Si and Ge this is often the dominant mechanism since they are “indirect band gap semiconductors.” Physically this means that the minimum energy gap between $E_c$ and $E_v$ does not occur at the same point in the momentum space.

Since momentum ($k$) must be conserved in any energy level transition, GaAs can easily make direct transitions which makes it useful for LEDs. $E_p = h\nu = E_g$. While Si and Ge tend to recombine through intermediate trapping levels.
Indirect Recombination

Let us consider the following example where an impurity like Au is introduced that provides a “trapping level” or a set of allowed states at energy $E_t$.

The level $E_t$ is assumed to act like an acceptor, that is, it can be neutral or negatively charged. Recombination is accomplished by trapping an e- and a hole. (The analysis can be easily extended to the case where the trap acts like a donor, i.e. + and neutral charge states).

- “1” = e- capture
- “2” = e- emission
- “3” = hole capture
- “4” = hole emission

$1 + 3$ or $3 + 1 \equiv$ recombination
Indirect Recombination

The probability of each of these events occurring is governed by FD statistics i.e. the position of the Fermi level. Note that $E_F$ is ONLY VALID UNDER EQUILIBRIUM ($np = n_i^2$) CONDITIONS. Let us consider the equilibrium situation first.

$$f_t = \frac{1}{1 + e^{\frac{E_t - E_F}{kT}}}$$

Probability $e^-$ occupies trap levels

$1 - f_t = \text{Prob. } e^- \text{ does not occupy trap level} = \text{Prob. hole occupies}$.

Therefore,

$$N_t f_t = \text{number of occupied centers} \quad (N_t = \text{concentration of centers})$$

$$N_t (1 - f_t) = \text{number of empty centers}$$
Indirect Recombination

We assume that the capture rate of a center is proportional to the number of $e^-$ (n) in the CB and to the number of empty trapping centers $N_t(1 - f_t)$. Then,

$$R_1 = v_{\text{the}} \sigma_n n N_t (1 - f_t) \quad (42)$$

where $v_{\text{the}} = \text{thermal velocity of } e^- (\approx 10^7 \text{ cm/sec})$

$\sigma_n = \text{capture cross-section for } e^- (\approx 10^{-15} \text{ cm}^2)$

$n = \text{number of } e^- \text{ in CB}$

Similarly,

$$R_2 = e_n N_t f_t \quad (43)$$

where $e_n = \text{emission probability}$

$N_t f_t = \# \text{ of filled trapping centers}$
Indirect Recombination - SRH

For holes, we can write similar equations:

\[ R_3 = v_{thp} \sigma_p n_t f_t \]  \hspace{1cm} (44)  
\[ R_4 = e_p n_t (1 - f_t) \]  \hspace{1cm} (45)

Assuming, \( v_{the} \sigma_n = v_{thp} \sigma_p \), we can show from (42) - (45) that the net recombination rate is given by:

\[
U = \frac{v_{th} \sigma_n N_t (pn - n_i^2)}{n + p + 2n_i \cosh \left[ \frac{E_t - E_i}{kT} \right]}
\]  \hspace{1cm} (46)

This is an extremely important result and is based on the work of Shockley, Read, and Hall. This theory is known as SRH recombination.
Indirect Recombination - SRH

\[ U = \frac{\nu_{th} \sigma_n N_t (p - n^2)}{n + p + 2n_i \cosh \left[ \frac{E_t - E_i}{kT} \right]} \]  \hspace{1cm} (46)

From (46) we observe:

- the “driving force” or the rate of recombination is \( \propto np - n_i^2 \) (that is, the deviation from the equilibrium condition).

- \( U = 0 \) when \( np = n_i^2 \).

- \( U \) is maximum when \( E_t = E_i \). That is, trapping levels near the mid-band are the most efficient recombination centers. When \( E_t \to E_c \)
  \( \Rightarrow e^- \) capture is more probable while hole capture is less probable.
  \( \Rightarrow e^- \) emission back to conduction band is highly probably.

Therefore, the recombination rate falls.
Indirect Recombination: N-type Semiconductors

Let us assume that $E_t = E_i$ so that

$$U = v_{th} \sigma N_t (pn - n_i^2) / [n + p + 2n_i] \quad (47)$$

a) N-type: $n >> p$ and $n >> n_i$, then from (47)

$$U \cong v_{thp} \sigma_p N_t (pn - n_i^2) / n$$

Now, $n_i^2 = n_{no} p_{no}$ (equilibrium values)

and $n \cong n_{no}$ (low level injection)

∴ $U \cong v_{thp} \sigma_p N_t (p n_{no} - n_{no} p_{no}) / n_{no}$

$$U \cong v_{thp} \sigma_p N_t (p - p_{no}) \quad (48)$$

But we defined $U$ as: $U = \Delta p / \tau_p$

∴ $\tau_p = 1 / (v_{thp} \sigma_p N_t)$ \quad (49)

In N type material, lots of $e^-$ are around for capture. Therefore, the limiting factor in recombination is the minority carrier lifetime $\tau_p$. 
Indirect Recombination: P-type Semiconductors

b) **P-type**: \( p >> n \) and \( p >> n_i \)

\[
\therefore \tau_n = 1/(v_{thn} \sigma_n N_t) \quad (50)
\]

Again, the limiting factor is the minority carrier lifetime.

---

**N-type**

\( E_F \) near \( E_c \)

\[
\therefore E_t \text{ filled with } e^- \\
\therefore \text{ hole capture rate limits process} \\
\therefore \tau_p \text{ dominates}
\]

**P-type**

\( E_F \) near \( E_v \)

\[
\therefore E_t \text{ empty of } e^- \\
\therefore e^- \text{ capture rate limits process} \\
\therefore \tau_n \text{ dominates}
\]
Indirect Recombination

If the trap level is not at the middle of the bandgap \((E_t \neq E_i)\). Then for an **N type** semiconductor with \(n \equiv n_{no} \) and \(n_i^2 = n_{no}p_{no}'\), we find from expression (46):

\[
U = \frac{v_{th} \sigma N_t (p_n - p_{no})}{1 + \left(2n_i \over n_{no}\right) \cosh \left[\frac{E_t - E_i}{kT}\right]} = \frac{p_n - p_{no}}{\tau_r}
\]

where \(\tau_r\) is the recombination life time.

If \(pn < n_i^2\) when generation is taking place, we get from (46):

\[
G = -U = \frac{v_{th} \sigma N_t n_i}{2 \cosh \left[\frac{E_t - E_i}{kT}\right]} = \frac{n_i}{\tau_g}
\]

where \(\tau_g\) is the generation life time.

**In general,** \(\tau_r \neq \tau_g\). However, when \(E_t = E_i\), both \(\tau_r\) and \(\tau_g\) are minimized. This is observed from lifetime Vs. \((E_t - E_i)/kT\) plots.
C. Quasi-Fermi Levels

Under thermal equilibrium, we have:
\[ n = n_i e^{(E_F - E_i)/kT} \]
\[ p = n_i e^{-(E_F - E_i)/kT} \]

In non-equilibrium conditions such as (1) injection \((np > n_i^2)\) or (2) extraction \((np < n_i^2)\), we cannot use these relationships. Here, we define two new quantities called quasi-Fermi levels such that the relationships of the type shown above hold and:
\[ n = n_i e^{(E_{Fn} - E_i)/kT} \]
\[ p = n_i e^{-(E_{Fp} - E_i)/kT} \]

where \(E_{Fn} \equiv e^-\) Quasi-Fermi level; \(E_{Fp} \equiv\) hole Quasi-Fermi level. \(E_{Fn}\) and \(E_{Fp}\) are the mathematical tools; their values are chosen so that the “correct” carrier concentrations are given in non-equilibrium situations.

In general, \(E_{Fn} \neq E_{Fp}\)
Quasi-Fermi Levels

**Example:**

Let us consider a p-type semiconductor with \( N_A = 10^{16} \text{ cm}^{-3} \), \( \tau = 10 \mu\text{sec} \), and we shine light on it such that \( G_L = 10^{18} \text{ carriers cm}^{-3} \text{ sec}^{-1} \).

Here, \( p_o = N_A = 10^{16} \text{ cm}^{-3} \)

\[ \tau G_L = (10 \times 10^{-6} \text{ sec})(10^{18} \text{ carriers cm}^{-3} \text{ sec}^{-1}) = 10^{13} \text{ carriers cm}^{-3} \]

\[ n = n_o + \tau G_L = n_i^2/N_A + \tau G_L \approx 10^{13} \text{ cm}^{-3} \]

\[ p = p_o + \tau G_L = (10^{16} + 10^{13}) \approx 10^{16} \text{ cm}^{-3} \]

Then from (36) and (37) we get:

\[ E_{Fn} = E_i + kT\ln(n/n_i) \approx 0.18 \text{ eV above } E_i \]

\[ E_{Fp} = E_i - kT\ln(p/n_i) \approx 0.36 \text{ eV below } E_i \]
Quasi-Fermi Levels

\[ n \equiv 10^{13} \text{ cm}^{-3} \]

\[ p \equiv 10^{16} \text{ cm}^{-3} \]

Since \( p \equiv p_o \), \( E_{Fp} \) is essentially unchanged. \( E_{Fn} \) moves up to reflect \( n >> n_o \) (equilibrium value). Here,

\[ pn = n_i^2 \left( \frac{E_{Fn} - E_{Fp}}{kT} \right) \] (55)

In equilibrium: \( E_{Fn} = E_{Fp} = E_F \Rightarrow pn = n_i^2 \).

However, under injection:

\( E_{Fn} \) moves up towards the CB.

\( E_{Fp} \) moves down towards the VB.

Thus, \(|E_{Fn} - E_{Fp}|\) is a measure of the degree of non-equilibrium in the semiconductor.

We will use this concept of Quasi-Fermi levels in our discussion of PN junctions.
D. Non-Uniform Doping Distributions

In our discussion of semiconductor statistics, we have assumed spatially uniform doping levels. Often this is not the case as we can see for the Bipolar Transistors.

1. The impurity profile we have drawn represents the fixed positions of ionized donors and acceptors, i.e. $N_{D^+}$, $N_{A^-}$.

2. The carriers ($e^-$ and holes) are mobile and attempt to diffuse away from regions of high concentrations.
Non-Uniform Doping Distributions

Now, consider a simpler situation where we only have N-type material (no junctions) but \( N_D \neq \text{constant} \). Here,

1. Diffusion flux is given by:
   \[ F = D_n \frac{dn}{dx} \]

2. As the \( e^- \) move (drift) away, they leave behind + charged ions \( N_D^+ \) which try to pull \( e^- \) back.

- Equilibrium is established when 1 = 2. That is, diffusion and drift cancels. \( n(x) \) is not exactly the same as \( N_D(x) \) and a “built-in” electric field exists.

- The band diagram for this example can be constructed noting that \textbf{in equilibrium the Fermi level is a constant}. 
Non-Uniform Doping - Built-in $\varepsilon$-Field

Now, we know that in equilibrium:

\[ F_n = D_n \frac{dn}{dx} + \mu_n n \varepsilon \equiv 0 \]

\[ \therefore \varepsilon_x = -\left(\frac{D_n}{\mu_n n}\right)\frac{dn}{dx} \]

Substituting for \([D_n/\mu_n] = kT/q\) we get:

\[ \varepsilon_x = -(kT/q)(1/n)\frac{dn}{dx} \] (56)

Thus, the impurity gradient results in a built-in electric field in the semiconductor. Similarly, for holes:

\[ \varepsilon_x = (kT/q)(1/p)\frac{dp}{dx} \] (57)

\[ \therefore \text{The minority carrier } e^- \text{ injected into the base by the emitter will be speeded up in their travel across the base or p-region by the built-in } \varepsilon\text{-field.} \]
E. Basic Semiconductor Equations

• To this point we have discussed:
  – Carrier transport – *drift* and *diffusion*
  – Carrier generation
  – Carrier recombination.

• The mechanism which ties all of these together is the condition of continuity of current.

\[ J_p(x + \Delta x) - J_p(x) = \frac{\Delta x}{U} \]

\[ J_p = \text{hole current} \]
\[ U = \text{recombination rate} \]

From the conservation of charge,

\[ \text{Rate of change of number of holes in } \Delta x = \left( \text{holes recombining} \right) + \left( \text{net holes flowing out of } \Delta x \right) \]
Basic Semiconductor Equations

From our discussion of recombination,

\[ U = \frac{p - p_o}{\tau_p} \]

\[ \therefore \text{holes recombining} = U \Delta x = \frac{p - p_o}{\tau_p} \Delta x \]

Net holes flowing out of \( \Delta x \)

\[ = \frac{1}{q} J_p(x + \Delta x) - \frac{1}{q} J_p(x) \]

\[ = \frac{1}{q} \frac{dJ_p}{dx} \Delta x \]
Continuity Equations

Finally, the rate of change of holes within $\Delta x$

$$\frac{dp}{dt} \Delta x = \frac{1}{q} \frac{dJ_p}{dx} \Delta x + \frac{p - p_o}{\tau_p} \Delta x$$

$$- \frac{dJ_p}{q dx} \frac{1}{\tau_p} + \frac{p - p_o}{\tau_p} = - \frac{dp}{dt}$$

Similarly,

$$- \frac{1}{q} \frac{dJ_n}{dx} \frac{1}{\tau_n} + \frac{n - n_o}{\tau_n} = - \frac{dn}{dt}$$

Equations (58) and (59) are called the continuity equations and are fundamentally important in the operation of semiconductor devices.
Drift-Diffusion Equations

The following equations that we previously derived are the transport equations and are also fundamentally important in device operation.

\[ J_p = q \left( p \mu_p \varepsilon - D_p \frac{dn}{dx} \right) \quad (60) \]

\[ J_n = q \left( n \mu_n \varepsilon + D_n \frac{dn}{dx} \right) \quad (61) \]
Poisson’s Equation

One final equation is needed to describe device operation and can be obtained from Poisson’s equation:

\[ \frac{d\varepsilon}{dx} = \frac{\rho}{K\varepsilon_o} \]

Where \( \varepsilon = \) electric field
\( \rho = \) space charge density (charges/cm\(^3\))
\( K = \) dielectric constant
\( \varepsilon_o = \) permittivity of free space \( = 8.854 \times 10^{-14} \text{ F/cm} \)

Therefore,

\[ \varepsilon = \int \frac{\rho dx}{K\varepsilon_o} \]

If we know the charge density as \( f(x) \) we could find the electric field.
Poisson’s Equation

In general, the net charge density in a semiconductor is given by:
\[ \rho = q[p^+ + N_D^+ - n - N_A] \]

Furthermore,
\[ \varepsilon = -\frac{dV}{dx} \]

so that
\[ \frac{d^2V}{dx^2} = \frac{q}{K\varepsilon_o} [(n - p) + (N_A - N_D)] \] (62)

(58) – (62) constitute a complete set of equations to describe carrier, current, and field distributions. Given appropriate boundary conditions, we can solve them for an arbitrary device structure. Generally, we will be able to simplify them based on physical approximations.