ELEN 115  Problem Set 2

1. ED.13

Low pass STC

\( \tau = 100T \)  \( \tau \) Find \( V_o \) at \( t = T \)

\( \frac{dV_o}{dt} = \left| Pe^{-\frac{t}{\tau}} \right| \)  \( \text{Difference in slope between} \)

\( t = 0 \) \( \text{and} \) \( t = T \)

\( Pe^0 - Pe^{-100} = \sqrt{\frac{P - Pe^{-100}}{Pe^0}} \)

Expressed as percentage of slope at \( t = 0 \)

\( \frac{P - Pe^{-100}}{Pe^0} \)  \( 1 - e^{-100} = 0.01 = 1\% \)
2. P.D. 1.

Original circuit

\[ V_0 = V_{01} + V_{02} \]

Equal contribution of LP @ 0 Hz and HP @ \( \infty \) Hz

\[ LP @ DC \Rightarrow V_{OC1} = -V_I \left( \frac{R_2}{R_1 + R_2} \right) \]

\[ HP @ \infty Hz \Rightarrow V_{OC2} = V_I \left( \frac{C_1}{C_1 + C_2} \right) \]

\[ R_2 C_1 + R_2 C_2 = R_1 C_1 + R_2 C_2 \Rightarrow R_2 C_2 = R_1 C_1 \]

If the condition apply, we can say that:

\[ R_2 = R_1 = R \quad \text{and} \quad C_2 = C_1 = C \]

From original circuit, set up voltage divider

\[ V_0 = V_I \left[ \frac{R_2 C_2}{R_1 C_1 + R_2 C_2} \right] \]
Substitute $R$ for $R_1$ and $R_2$ and some for $C$.
3. Problem 2.

Refer to previous problem’s diagram.

\[ Z_1 = R_1 \parallel C_1 = R_1 \parallel \frac{1}{j\omega C_1} \]

\[ Z_2 = R_2 \parallel C_2 = R_2 \parallel \frac{1}{j\omega C_2} \]

Voltage divider.

\[ v_0 = v_1 \left[ \frac{Z_2}{Z_1 + Z_2} \right] = v_1 \left[ \frac{R_2}{1 + j\omega C_2 R_2} \right] \]

\[ \frac{R_2}{1 + j\omega C_2 R_2} = \frac{R_2}{1 + j\omega C_2 R_2} + \frac{R_1}{1 + j\omega C_1 R_1} \]

Since \( C_1 R_1 = C_2 R_2 = CR \)

\[ v_0 = v_1 \left[ \frac{R_2}{R_2 + R_1} \right] \]

\[ v_0 = v_1 \left( \frac{R_2}{R_2 + R_1} \right) \]

This is frequency independent because there are no terms with \( \omega \).
4. PD. 3.

\[ R_2 = 1 \text{ M} \Omega \]

\[ C_2 = 30 \text{ pF} \]

\[ \text{Oscilloscope} \]

\[ C_1 R_1 = C_2 R_2 = CR \]

We want 10 to 1 attenuation. Therefore,

\[ \frac{V_o}{V_i} = \frac{1}{10} \]

Recall relationship from previous problem

\[ \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1} \Rightarrow \frac{1}{1 \text{ M} \Omega + R_1} = \frac{1}{10} \]

\[ R_1 = 9 \text{ M} \Omega \]

\[ R, C_1 = R_2 C_2 \Rightarrow C_1 = \frac{30}{9} \text{ pF} \]

Input impedance \( Z_i + Z_2 \) (defined in last problem)

\[ \frac{R_1}{1 + j\omega CR} + \frac{R_2}{1 + j\omega CR} = \frac{10}{1 + j\omega CR} \]

This is 10 times higher than the input impedance of the oscilloscope \( \frac{1}{1 + j\omega CR} \)
5. PD.5

Voltage amplifier - gain

\[ A_{vo} = -100 \ \text{V/V} \]
\[ R_o = 0 \]
\[ R_i = 10 \ \text{k} \Omega \]
\[ C_i = 10 \ \text{pF} \]
\[ C_f = 1 \ \text{pF} \]
\[ R_s = 10 \ \text{k} \Omega \]

High frequency (cap short)

\[ V_o = 0 \ \text{V} \]

Low frequency (cap open)

\[ V_o = (-100) \left( \frac{1}{2} \right) V_s \]
\[ V_o = -50 V_s \]
\[ \frac{V_o}{V_s} = -50 \]

\[ 20 \log |50| = 33.98 \text{ dB} \]

\[ \gamma = \left( 10 \ \text{k} \Omega \parallel 10 \ \text{k} \Omega \right) \left( 11 \ \text{pF} \right) \]
\[ \tau = 55 \ \text{ns} \]
\[ \omega_o = \frac{1}{\tau} = \frac{1}{18 \ \text{MHz}} \]
Fig D.4(b)  
\[ V_I = 10 \text{ V step} \]
\[ R = 1 \text{ k}\Omega \]
\[ L = 1 \text{ mH} \]
\[ \zeta = \frac{L}{R} = 1 \text{ ms} \]

Low-pass  
\[ V_0 = 10(1 - e^{-\frac{t}{10\text{ ms}}}) \]

\[ V_0 \text{ is 10 at time} = 0^+ \]

Fig D.5(b)  
\[ V_I = 10 \text{ V step} \]
\[ R = 1 \text{ k}\Omega \]
\[ L = 1 \text{ mH} \]
\[ \zeta = \frac{L}{R} = 1 \text{ ms} \]

High-pass  
\[ V_0 = 10e^{-\frac{t}{10\text{ ms}}} \]
PD. 13.

$\zeta = 1 \text{ ms}$

10-V pulse (1 ms width)

$\zeta$ comparable to $T$

$\Delta V = \text{Undershoot High-pass Response.}$

$V_0 = 10 e^{-\frac{t}{\zeta}} \text{ (at } t = 1 \text{ ms)}$

$\Delta V = 10 - 10 e^{-\frac{1}{\zeta}} = \boxed{6.32 \text{ V}}$

Undershoot less than 1 V

$\Delta V \leq 1 \Rightarrow 1 \geq 10 - 10 e^{-\frac{1}{\zeta}}$

$\zeta \geq 9.49 \text{ ms}$
Fig D.7(a)  $C_f = 100 \text{ pF}$

Find $R$ so that gain has $f_0$ of 1 kHz.

From Example D.4. (equations not derived here)

$$\frac{V_o}{V_s} = \frac{M}{1 + j\omega RC_f(-M + 1)} \Rightarrow \omega = RC_f(-M + 1)$$

We want $f_0 = 1 \text{ kHz} \Rightarrow \omega_0 = 2\pi f_0 = 6283 \text{ rad/s}$

$$\omega = \frac{1}{c_{283}} = (R)(100 \times 10^{-12})(100 + 1)$$

$$R = 15.758 \text{ } \Omega \approx 16 \text{ k } \Omega$$

2 resistors 4.7 kΩ and 10 kΩ

Voltage divider, 15 V supply → 10 V output

\[ V_o = (15) \left( \frac{10}{14.7} \right) \]

\[ V_o = 10.2 \text{ V} \]

We want output of 10.00 V
To get the fraction smaller, we have to shunt the 10 kΩ resistor

\[ 10.00 = 15 \left( \frac{R_{eq}}{R_{eq} + 4.7} \right) \Rightarrow R_{eq} = 9.4 \text{ kΩ} \]

10 kΩ \[ R = 9.4 \Rightarrow R = 157 \text{ kΩ} \]

We want 3.33 kΩ for \( R_0 \)

\[ R_0 = 4.7 \text{ kΩ} \parallel 9.4 \text{ kΩ} \Rightarrow R_0 = 3.13 \text{ kΩ} \]

To get 3.33 kΩ, we can put a resistor of 200 Ω in series to make \( R_0 = 3.33 \text{ kΩ} \)

We want 10.00 V and 3.00 kΩ

\[ 10 = 15 \left( \frac{10 \parallel R_1}{4.7 \parallel R_1 + 10 \parallel R_1} \right) \quad \text{* Algebra not shown} \]

We must shunt the 4.7 kΩ with 157 kΩ and shunt the 10 kΩ with 90
We want \( \frac{1}{3} I \) across \( R \)

We must shunt the load resistor with another resistor

Voltage is same across the 2nd resistors. We use Ohm's Law:

\[
\frac{\frac{1}{3}V}{R} = \left( \frac{\frac{1}{3}V}{R} \right)(R_1)
\]

\[
R_1 = \frac{R}{2}
\]

\[
R_{\text{in}} = \frac{R}{2} || R = \frac{R}{2} + \frac{R}{2}^{-1}
\]

\[
R_{\text{in}} = \left( \frac{\frac{1}{3}V}{R} + \frac{1}{R} \right) = \frac{R}{3}
\]

\( R_1 \) is 10% too high \( \Rightarrow (1.1)R_1 \)

We can add a shunt resistor to make \( R_1 \) 10% smaller

\[
\frac{1}{R_S} + \frac{1}{1.1R} = \frac{1}{2}
\]

\[
(1.1)R + 2R_S
\]

\[
0.2R_S = 1.1R \Rightarrow R_S = 5.5R
\]
2) We can put another resistor in series with the load, to compensate for the 10% increase of the $R_1$ value. We want $\frac{1}{3}$ of $I$ to go through the load, because $R_1$ is 10% larger, we must make $R$ large enough that the division ratio is still the same.

So, we want: 

$$\left(\frac{R}{3}\right)(x)(1.1) = R + R_s$$

$$R_s = 0.1R$$
Thevenin (left side)
\[ V_{th} = (9) \left( \frac{1.2}{2.2} \right) = 4.909 \text{ V} \]
\[ R_{th} = 11 \ \Omega = 0.545 \ \text{k}\Omega \]

Thevenin (right side)
\[ V_{th} = (9) \left( \frac{11}{20.1} \right) = 4.925 \text{ V} \]
\[ R_{th} = 9.1 \ \Omega = 4.98 \ \text{k}\Omega \]

Current
\[ \frac{4.925 - 4.909}{8.525} = 1.88 \mu\text{A} \]

Voltage
\[ (1.88 \times 10^{-3})(3) = 5.64 \text{ mV} \]
P1.18

Note: This circuit is symmetrical. Therefore, the current $I_x$ will be split in half. Because the current each side is the same, there is nothing across $R_5$ because they will cancel each other out.

On each side, we have: $I_x = \frac{V_x}{2k\Omega}$

Thus $R_{eq} = \frac{V_x}{I_x} = \frac{2k\Omega}{2} = 1k\Omega$

If $R_4 = 1.2\ k\Omega$, there is no longer symmetry. So, we use Thévenin like the previous problem.

$0.5k\Omega$ $1k\Omega$ $0.545k\Omega$ (calculation not shown)

$0.5V_x$
Current across $R_5$.

\[ I_5 = \frac{0.545V_x - 0.5V_x}{2.045} = 0.022V_x \]

* Voltage at node 1.

\[ V_1 = 0.5V_x + (I_5)(0.5) \]
\[ = 0.5V_x + (0.022V_x)(0.5) = 0.511V_x \]

* Voltage at node 2.

\[ V_2 = 0.511V_x + (0.022V_x)(1) \]
\[ = 0.533V_x \]

* Current on left side

\[ I_1 = \frac{V_x - 0.511V_x}{1} = 0.489V_x \]
\[ I_2 = \frac{V_x - 0.533V_x}{1} = 0.467V_x \]

\[ I_x = 0.489V_x + 0.467V_x = 0.956V_x \]

\[ R_{eq} = \frac{V_x}{I_x} = \frac{1}{0.956} = 1.05 \text{ k}\Omega \]