Decentralized dynamic output feedback for robust stabilization of a class of nonlinear interconnected systems

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Abstract

The objective of this paper is to propose an approach to robust stabilization of systems which are composed of linear subsystems coupled by nonlinear time-varying interconnections satisfying quadratic constraints. The proposed algorithms, which are formulated within the convex optimization framework, employ linear dynamic feedback structure involving local Luenberger-type observers. It is also shown how the new methodology can produce improved results if interconnections have linear parts that are known a priori. Examples of output stabilization of inverted pendulums and decentralized control of a platoon of vehicles are used to illustrate the applicability of the design method.

Keywords: Nonlinear interconnected systems; Decentralized dynamic output feedback; LMIs; Robustness; Control of platoons of vehicles

1. Introduction

With the emergence of the powerful convex optimization toolboxes involving linear matrix inequalities (LMIs), solving problems of controller design within the convex optimization framework became very attractive (e.g. Boyd, El Ghaoui, Feron, & Balakrishnan, 1994; Dullerud & Paganini, 2000; Gahinet & Apkarian, 1994; Gahinet, Nemirovski, Laub, & Chilali, 1995; Garcia, Daafouz, & Bernussou, 2002; Ho & Lu, 2003; Iwasaki & Skelton, 1994; Scorletti & Duc, 2001). Of wide-spread interest have been the control problems of formulating sufficient conditions for computing output feedback control laws using convex optimization methods due to the fact that the necessary and sufficient conditions are known to be nonconvex, in general. These problems become increasingly more difficult to solve when decentralized information structure constraints are imposed in the control design (Cao, Sun, & Mao, 1998; Geromel, Bernussou, & de Oliveira, 1999; Geromel, Bernussou, & Peres, 1994; Qi, Salapaka, Voulgaris, & Khammash, 2004; Šiljak & Zečević, 2004; Yang & Wang, 1999; Zečević & Šiljak, 2004). These information structures can be found in important applications, such as power systems (Zečević, Nešković, & Šiljak, 2004), control of formations of unmanned vehicles (Stanković, Stanojević, & Šiljak, 2000) and control of large structures (Li, Kosmatopoulos, Yoannou, & Ryciotaki-Boussalis, 2000), to name few.

In this paper we propose novel sufficient conditions for the design of decentralized dynamic output controllers in the convex optimization context for stabilization of interconnected systems with linear subsystems and nonlinear time-varying interconnections. Controllers are designed to guarantee robust stability of the overall system and, in addition, maximize the bounds of unknown interconnection terms, starting from the methodology proposed in Šiljak and Stipanović (2000). In contrast to the approach in Stanković and Šiljak (2006), where general dynamic controller structure is treated, we shall adopt here the controller structure containing local observers of Luenberger type. Several algorithms are proposed in the general
case of full order observers, differing by complexity and the degree of interdependence between the observer and the feedback gains, where no additional constraints on the parameters of the system model are imposed (see Pagilla & Zhu, 2005; Šiljak & Stipanović, 2001). It is also shown how the proposed scheme can be used to build reduced-order observers. Particular attention is paid to the case when linear parts of interconnections are known a priori, and an algorithm is proposed which takes advantage of this knowledge to come up with improved results. To illustrate the application of the proposed schemes we include two examples, the first dealing with interconnected pendulums, and the second with the problem of platoons of vehicles in the case when the velocity and acceleration of the neighboring vehicles are not accessible.

2. Problem statement

Consider a nonlinear interconnected system $S$ composed of a finite number $N$ of subsystems represented by

$$S_i : \dot{x}_i = A_{ii}x_i + B_{ii}u_i + h_i(t, x), \quad y_i = C_{ii}x_i, \quad i = 1, \ldots, N, \quad \sum_{i=1}^{N} n_i = n, \quad \sum_{i=1}^{N} p_i = p,$$

where $x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{p_i}$ and $y_i \in \mathbb{R}^{m_i}$ are the subsystem state, input and output vectors, respectively, $x = (x_1^T, \ldots, x_N^T)^T$ and $h_i(t, x) : \mathbb{R}^{n+i} \rightarrow \mathbb{R}^{m_i}$ are piecewise continuous vector functions in both arguments, satisfying in their domains of continuity the following quadratic inequalities:

$$h_i(t, x)^T H_i h_i(x), \quad i = 1, \ldots, N,$$

$$z_1 > 0$$

where $z_1 > 0$ are bounding parameters and $H_i$ constant $l_i \times n$ matrices, $i = 1, \ldots, N$ (see, e.g. Šiljak & Stipanović, 2000, 2001). The entire interconnected system $S$ can be represented in a compact form as

$$S : \dot{x} = Ax + Bu + h(t, x), \quad y =Cx, \quad \text{Subject to} \quad X_f > 0,$$

$$\text{Minimize} \quad \text{Tr} \Gamma \quad \text{Subject to} \quad X_f > 0, \quad \begin{bmatrix} X_f A_f + A_f^T X_f & X_f & H_f^T \\ X_f & -I & 0 \\ H_f & 0 & -\Gamma \end{bmatrix} < 0.$$

However, in general, dynamic output feedback design cannot be done directly using (9), since the second matrix inequality is not an LMI in both $X_f$ and the feedback parameter matrix. In the case of static state feedback the problem can be reformulated into an LMI problem by a simple change of variables (Geromel et al., 1994). In the case of dynamic output feedback the problem becomes far more complex. A decoupled quadratic Lyapunov function with block-diagonal weighting matrix has been used in Šiljak and Stipanović (2001) to determine the dynamic controller parameters. However, the proposed design procedure imposes additional constraints on the system model characteristics. Thus, in the next section we will provide several modifications of (9) obtained by convexifying the constraints. Solutions to these problems will provide guaranteed feasible solutions to (9) and the upper bound of the objective function $\text{Tr} \Gamma$. This is implied by the fact that these convex conditions are more restrictive than the nonconvex conditions in (9).
3. Robust decentralized controller design

3.1. General case—full order observer

Define $Q = \text{diag}\{Q_1, \ldots, Q_N\}$, $P = \text{diag}\{P_1, \ldots, P_N\}$, $W = \text{diag}\{W_1, \ldots, W_N\}$ and $V = \text{diag}\{V_1, \ldots, V_N\}$, where $Q_i$, $P_i$, $W_i$ and $V_i$ are $n_i \times n_i$, $n_i \times n_i$, $m_i \times n_i$ and $n_i \times p_i$ matrices, respectively. Also, for compactness of notation let us define

$$\Phi(S, L, M, \Gamma) = \begin{bmatrix} S & L & M \\ L^T & -I & 0 \\ M^T & 0 & -\Gamma \end{bmatrix},$$

where $S$, $L$, $M$ and $\Gamma$ are matrices of appropriate dimensions.

**Problem 1.** Minimize $\text{Tr} \, \Gamma$

Subject to $Q > 0$, $P > 0$, $\Phi(S_1, L, QH^T, \Gamma) < 0$, $\Phi(S_2, P, -H^T, \Gamma) < 0$,

where $S_1 = AQ + QA^T + BW + W^TB^T$ and $S_2 = PA + A^TP - VC - C^T V^T$.

**Theorem 1.** System $S$ is robustly stabilized by the controller $F$ if Problem 1 is feasible. The controller parameters are given by $K = WQ^{-1}$, $L = P^{-1}V$.

**Proof.** It will be shown that there exists a real number $\lambda > 0$ such that the matrix $X_f = \text{diag}\{\lambda^2 - Q, P\}$ satisfies LMIs (9) for some $\Gamma > 0$, where $P$ and $Q$ are solutions of Problem 1 (see Boyd et al., 1994 for a similar approach to the basic stabilization problem). Introducing (7) and $X_f$ into (9), one obtains

$$\begin{bmatrix} \lambda S_1 & -LC & I & 0 & \lambda QH^T \\ -C^T L^T & S_2 & 0 & P & -H^T \\ I & 0 & -I & 0 & 0 \\ 0 & P & 0 & -I & 0 \\ \lambda HQ & -H & 0 & 0 & -\Gamma \end{bmatrix} < 0. \tag{13}$$

Applying the Schur’s complement formula after convenient column and row permutations, we obtain the following conditions equivalent to (13):

$$\begin{bmatrix} 1 & 0 \\ 0 & -LC \end{bmatrix}, \begin{bmatrix} 0 & -H \end{bmatrix}^T,$$

where $\Xi_1 = \begin{bmatrix} -I & P \\ P & S_2 \end{bmatrix}$, $\Xi_2 = \begin{bmatrix} 0 & -LC \\ 0 & -H \end{bmatrix}$, $\Xi_3(X) = \begin{bmatrix} S_1 & QH^T \\ H & Q \\ -X \end{bmatrix}$ (with $X$ denotes an $n \times n$ matrix), and $\Gamma = \lambda^2 - Q$.

Let $\nu = \lambda \max(\Xi_3^{-1})$, $a = \lambda \max(\Xi_1^{-1})$, and $\mu = \lambda \max(\Xi_3(I_0))$, where $I_0 = \text{diag}\{I_{11}, \ldots, I_{NN}\}$ is the optimal $\Gamma$ obtained by solving Problem 1. Both eigenvalues $\mu$ and $\nu$ are negative, since $\Xi_1$ and $\Xi_3(I_0)$ represent principal minors of the negative definite matrices $\Phi(S_1, I, QH^T, \Gamma)$ and $\Phi(S_2, P, -H^T, \Gamma)$. Choose $\Gamma = \nu \Gamma_0$, where $\nu > |\theta| / |\mu|$, $\theta = -1 + a\nu$, and assume that $0 < \lambda < \lambda^*$; then, we have

$$\lambda_{\max}(\Xi_3(\Gamma)) = \lambda_{\max}(\Xi_3((\lambda^*/\lambda)\Gamma)) \leq \lambda_{\max}(\Xi_3(\Gamma_0)) = \mu,$$

having in mind that $\lambda^*/\lambda > 1$. For this choice of $\Gamma$ and $\lambda$, (14) is implied by

$$\mu \lambda - \theta < 0, \tag{15}$$

which is true for $|\theta| / |\mu| < \lambda < \lambda^*$. Therefore, the required $\lambda$ exists, and we have the result. □

**Remark 1.** The local robustness degrees $\zeta_i = 1/\sqrt{\gamma_i^2(\theta/|\mu|)}$, $i = 1, \ldots, N$, guaranteed on the basis of Theorem 1 are too conservative. More realistic values can be obtained by introducing into (9) the controller parameters obtained by (12), and by solving the corresponding minimization problem with variables $X_f$ and $\Gamma$, as will be done in all the examples presented.

**Remark 2.** Problem 1 implements, in fact, the separation principle. The constituent problems $Q > 0$, $\Phi(S_1, I, QH^T, \Gamma) < 0$ and $P > 0$, $\Phi(S_2, P, -H^T, \Gamma) < 0$ can be solved independently, the first providing $K$ as in the state feedback design (Šiljak & Stipanović, 2000), and the second $L$, robustly stabilizing the observer, so that $\Gamma = \text{diag}\{\max(\gamma_i^{1,2})I_{11}, \ldots, \max(\gamma_i^{N,N})I_{NN}\}$.

**Remark 3.** The proof of Theorem 1 gives the idea to simplify LMIs in (11) as follows:

**Problem 2.**

Minimize $\text{Tr} \, \Gamma$

Subject to $Q > 0$, $P > 0$, $\Xi_3(\Gamma) < 0$, $\Xi_1 < 0$. \tag{16}

Controller parameters are obtained by using (12). The achievable robustness degree is, in general, lower than the one obtained by solving Problem 1. Namely, it is possible to show using the methodology of Theorem 1 that if $Q_0$, $W_0$ and $\Gamma_0$ are obtained by solving Problem 2, then there exist $\rho > 0$ and $\beta > 1$ such that $\Phi(\rho(AQ_0 + Q_0A^T + BW_0 + W_0B^T), I, \rho Q_0H^T, I, \rho \Gamma_0) < 0$.

The structure of (13) gives an idea to construct more complex algorithms, trying to get higher robustness degree by taking into account the interdependence between $K$ and $L$ in the LMIs (9).

**Problem 3.**

Step 1:

Minimize $\text{Tr} \, \Gamma$

Subject to $P > 0$, $\Phi(S_2, P, -H^T, \Gamma) < 0$. \tag{17}

Step 2:

Minimize $\text{Tr} \, A$

Subject to $Q > 0$,

$$\begin{bmatrix} S_1 & I & -LC & 0 & QH^T \\ I & -I & 0 & 0 & 0 \\ -C^T L^T & 0 & S_2 & P & -H^T \\ 0 & 0 & I & P & -I \\ H & Q & 0 & -H & 0 & -\Gamma A \end{bmatrix} < 0, \tag{18}$$
Theorem 2. System S is robustly stabilized by the controller F if Problem 3 is feasible. Controller parameters are given by (12). The robustness degree bounds are given by $z_l = 1/\sqrt{h_l \delta_l}$.

Proof. The proof follows simply from the fact that the second inequality in (18) is identical to inequality (9) for $X_f = \text{diag}(Q^{-1}, P)$, with $\Gamma$ replaced by $\Gamma A$. Notice that Steps 1 and 2 have to be performed consecutively and not simultaneously, like in Problems 1 and 2. □

Remark 4. Notice that the exposed methodology gives rise to similar algorithms which could be derived from alternative realizations of the closed-loop model (6). One can take, for example, $z = (z'_1, z'_2)^T$, $z_1 = x$, $z_2 = x - w$, $A_f = \begin{bmatrix} A + BK & -BK \\ 0 & A - LC \end{bmatrix}$, $C_f = [C_1, 0]$ and $h_f(t, z) = \begin{bmatrix} h(z'_1)^T; h(z'_2)^T \end{bmatrix}^T$, and come up with a problem similar to Problem 3, in which $K$ is determined in Step 1, and $L$ in Step 2.

3.2. General case—reduced order observer

The results of the preceding paragraph can be directly extended to the design of controllers with decentralized reduced order observers. Assume that $C_i = \begin{bmatrix} 0_{(n_i-p_i)\times n_i} & P_i \end{bmatrix}$, $P_i \leq n_i$; if $x_i$ is decomposed as $x_i = (x'_i)^T, (x''_i)^T$, where $x'_i \in \mathbb{R}^{n_i-p_i}, x''_i \in \mathbb{R}^{p_i}$, then $y_i = x''_i$ and the output of the local reduced order observer is an estimate of $x'_i$. Assume also that the local dynamic controllers $F_i$ have the following form (Kwakernaak & Sivan, 1972):

\[
\dot{y}_i = A_{i1}^{11} y_i + A_{i1}^{12} y_i + B_{i1}^{1} u_i + L_i (y_i - A_{i2}^{11} y_i - A_{i2}^{12} y_i - B_{i2}^{1} u_i),
\]

\[
u = G_i y_i + J_i y_i = K_i \hat{z}_i,
\]

where $A_i = \begin{bmatrix} A_{i1}^{11} & A_{i1}^{12} \\ A_{i2}^{11} & A_{i2}^{12} \end{bmatrix}$, $B_i = \begin{bmatrix} B_{i1}^{1} \\ B_{i2}^{1} \end{bmatrix}$ $(A_{i1}^{11}$ and $B_{i1}^{1}$ are $p_i \times p_i$ matrices and $A_{i2}^{12}$ and $B_{i2}^{1}$ are $p_i \times p_i$ matrices, respectively), $K_i = [G_i; J_i]$ and $\hat{z}_i = (y'_i, y''_i)^T = (y'_i, y''_i)^T$ (differentiation of $y_i$ in (19) can be avoided by standard transformation of variables). Defining $\eta_1 = \dot{y}_i - x''_i$, $\hat{z}_i = (\hat{z}_1, \ldots, \hat{z}_N)^T$ and $\eta = (\eta_1, \ldots, \eta_N)^T$, we take $\hat{z} = (\hat{z}_1, \eta_N)^T$ as a new state vector for $S_f = (S, F)$, and obtain

\[
\dot{\hat{z}} = \begin{bmatrix} A + BK & \hat{L} A_{12} A_{21}^T \\ 0 & A_{11} - L A_{21}^{12} \end{bmatrix} \hat{z} + h_f(t, z),
\]

with the decomposition $h_f(t, z) = (h_1'(x)^T, h''_1(x)^T)^T$ induced by the decomposition of $x_i$ into $x'_i$ and $x''_i$, so that

\[
\dot{h}_f(t, z)^T h_f(t, z) \leq z^T \tilde{H}_f^T \tilde{H}_f z,
\]

where $\tilde{H}_f = \begin{bmatrix} H_f & 0 \\ H_f^T & 0 \end{bmatrix}$, while $\tilde{H}_f = \text{diag}(\bar{h}_1, \ldots, \bar{h}_N)$, \(\bar{h}_i = \text{diag}(\dot{y}_i, y''_i)^T\), while $\bar{H}_f$ is an $l_i \times (n_i - p_i)$ matrix containing the first $n_i - p_i$ columns of $H_i$, having in mind that $H_i x = \bar{H}_i \xi - \bar{H}_i \eta$.

The structure of the closed-loop model (21) shows that controller design can be entirely based on the methodology exposed in the first paragraph. Problem 1 and Theorem 1 give rise to

Corollary 1. The system $S$ in which $C = \text{diag} \{0_{(n_1-p_1)\times p_1}, \ldots, 0_{(n_N-p_N)\times p_N}\}$, $p_i \leq n_i, i = 1, \ldots, N$, is robustly stabilized by the dynamic controller $F$ defined by (19), (20) if the following problem is feasible:

Minimize $\text{Tr} \Gamma$

Subject to $Q > 0, ~ \bar{P} > 0, \Phi(S_1, I, QH^T, \Gamma) < 0, \Phi(\bar{S}_2, \bar{P}, -H^T, \Gamma) < 0$

with $\bar{S}_2 = \bar{P} A_{11}^{11} + (A_{11}^{11})^T \bar{P} - \bar{V} A_{21}^{12} - (A_{21}^{12})^T \bar{V}^T, \bar{P} = \text{diag}(P_1, \ldots, P_N)$ and $\bar{V} = \text{diag}(\bar{V}_1, \ldots, \bar{V}_N)$ ($P_i$ and $\bar{V}_i$ are $(n_i - p_i) \times (n_i - p_i)$ and $(n_i - p_i) \times p_i$ matrices, respectively). Controller parameters are obtained by using (12).

3.3. A linear part of interconnections known

Known linear interconnections between the subsystems $S_i$ in $S$ can be represented by a full $n \times n$ matrix $A_\delta$ containing off-diagonal interconnection blocks, so that $A + A_\delta$ becomes the new state matrix in the linear part of $S$ in (2); function $h(t, x)$ still represents the unknown part of interconnections. The design methodology proposed in Sections 3.1 and 3.2 can be extended to this case, trying to take advantage of the additional a priori information. Notice, however, that by replacing $A$ by $A + A_\delta$ in the observer equation for $F$ in (5) one violates the adopted information structure constraint, i.e. the dynamic controller ceases to be decentralized. Inserting $A + A_\delta$ only in the state model (3), we obtain $A_f = \begin{bmatrix} A + BK & -LC \\ -A_\delta & A + A_\delta - LC \end{bmatrix}$.

This fact indicates that the design scheme could now be based on the problems described in Sections 3.1 and 3.2 by inserting the new information in the form of $A_\delta$ at the corresponding places in the related LMIs. However, robust stabilization is achievable when the interconnections are sufficiently weak. For example, Problem 1 gives rise to:

Problem 4.

Minimize $\text{Tr} \Gamma$

Subject to $P > 0, Q > 0, ~ \Phi(S_1, I, QH^T, \Gamma) < 0, \Phi(S_2, P, -H^T, \Gamma) < 0$

with $S_{2\delta} = P(A + A_\delta) + (A + A_\delta)^T P - VC - C^T V^T$. 

Where $A = \text{diag}(\delta_1 I_1, \ldots, \delta_N I_N)$ ($\delta_i > 0$), while $P, S_2, \Gamma$ and $L = P^{-1} V$ are obtained in Step 1.
Theorem 3. The system $S$ with known linear interconnections (modelled by adding $A_\delta$ to $A$ in (3)) is robustly stabilized by the decentralized dynamic controller $F$ in (5) if Problem 4 is feasible and

$$\delta < \frac{\mu^2}{8\delta y_\delta \lambda P}\lambda Q, \quad (25)$$

where $\delta = \lambda_{\text{max}}(A_0^T A_\delta)$, $\lambda P = \lambda_{\text{max}}(P^2)$, $\lambda Q = \lambda_{\text{max}}(Q^2)$, $y_\delta = \lambda_{\text{max}}(\Sigma_1^T)$, matrix $\Sigma_1$ is obtained from $\Sigma_1$ in (14) by replacing $S_2$ with $S_2\delta$, and $\theta_\delta = -1 + 2ay_\delta$.

Proof. The proof is based on a line of thought similar to that applied in Theorem 1. Inserting $X_f = \text{diag}[\lambda^{-1} Q^{-1}, P]$ and

$$A_f = \begin{bmatrix} A + BK & -LC \\ -A_\delta & A + A_\delta - LC \end{bmatrix}$$

into (9) we obtain

$$\begin{bmatrix} \lambda S_1 & -L_\delta \\ -L_\delta^T & -I \end{bmatrix} - \begin{bmatrix} 0 & \lambda Q \lambda H \\ \lambda H Q & -H \\ I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} < 0, \quad (26)$$

where $L_\delta = LC + \lambda Q A_1^T \delta P$. The last inequality is equivalent to $\Sigma_1 < 0$ and

$$\lambda \Sigma_2(\Gamma_\delta) - (\Sigma_2 + \lambda \Sigma_20) \Sigma_1^{-1} (\Sigma_2 + \lambda \Sigma_20)^T + \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} < 0, \quad (27)$$

where $\Sigma_20 = \begin{bmatrix} 0 & -Q A_1^T \delta P \\ 0 & 0 \end{bmatrix}$. Assume, similarly as in Theorem 1, that $\Gamma = \lambda^* \Gamma_0$ for some $\lambda^* > 0$, where $\Gamma_0$ is the optimal value obtained by solving Problem 4. Assume that $0 < \lambda < \lambda^*$. Then, (27) is implied by

$$-2\delta y_\delta \lambda P \lambda^2 + \mu \lambda - \theta_\delta < 0, \quad (28)$$

having in mind that $\lambda_{\text{max}}(\Sigma_2(\Gamma_\delta)) \leq \lambda_{\text{max}}(\Sigma_2(\Gamma_0)) = \mu$; notice that $y_\delta < 0$ by assumption, as a consequence of the feasibility of Problem 4. The existence of $\lambda > 0$ satisfying (28) is guaranteed if (25) holds, since then we have $D = \mu^2 - 8\delta y_\delta \lambda P \lambda Q > 0$. Consequently, we choose

$$\frac{-\mu - \sqrt{D}}{4\delta y_\delta \lambda P \lambda Q} = \lambda_1 < \lambda^* \leq \lambda_2 = \frac{-\mu + \sqrt{D}}{4\delta y_\delta \lambda P \lambda Q},$$

where $0 < \lambda_1 < \lambda_2$ since $\mu < 0$ and $\sqrt{D} \leq |\mu|$, so that $\lambda$ can take any value in the interval $[\lambda_1, \lambda^*]$. Thus, the result. \qed

The local guaranteed robustness degree bounds are now $\xi_i = 1/\sqrt{\lambda_i^* \lambda_i}, i = 1, \ldots, N$.

4. Examples

4.1. Inverted pendulums

The first example illustrates how the proposed methodology can be used to obtain connective stability (Šiljak, 1991) and robustness with respect to modelling errors in the case when a system is not stabilizable using static output feedback.

Consider the motion of two inverted pendulums connected by a spring which can slide up and down the rods of the pendulums in jumps of unpredictable size and direction between the support and the height equal to 1 (Šiljak & Stipanović, 2000). A linearized and normalized model is given by

$$\mathbb{S} : \dot{x} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u + h(t,x), \quad (29)$$

$$y = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

where $h(t,x) = e(t,x)Gx$, $G = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ and $e(t,x) : \mathbb{S} \rightarrow [0, 1]$ represents a normalized interconnection parameter. We want to compute a decentralized control law which would connectively stabilize the system for all values of $e(t,x) \in [0, 1]$.

The decentralized static state feedback provides $\pi = 4.4950$ with the local controller gain matrix $K = [-725.9085 - 40.4346]$ and the corresponding closed-loop poles $z = \{-20 \pm j17.8093\}$. Clearly, the system is not stabilizable by static output feedback, since two coefficients of the characteristic equation remain fixed to zero irrespective of the controller parameters.

Table 1 gives robustness degree $\pi$ provided by dynamic output feedback obtained by the proposed algorithms. Case A corresponds to the situation in which $H = G$ in the three algorithms from Section 3.1, Case B to $H = G$ with $A_\delta$ applied (derived from Problems 1–3 in accordance with the methodology of Problem 4 and Theorem 3), and Case C to the situation with no a priori knowledge, when $H = I$ and $A_\delta = 0$, and the algorithms from Section 3.1 are applied. It is possible to conclude that the best results are obtained by solving Problem 3; the worst case corresponds to Problem 2, as expected. Notice that in Case C none of the algorithms ensures connective stability. For Problem 2, connective stability is achieved only in Case B, when the information about the interconnections is included (in Case B we have, in fact, $e(t,x) = 0.5 + e'(t,x)$, where $e'(t,x) \in [-0.5, 0.5]$, so that any value of $\pi > 0.5$ is sufficient for connective stability). All values of $K$ and $L$ and the corresponding modes are not presented because of the lack of space. For example, for Problem 1 and Case A we
have \( K_i = [-79.1666, -11.2883] \), \( L_i^T = [27.7711, 15.7991] \), with local closed-loop poles (\(-27.2275, -0.5435, -0.5441 \pm j6.8052\)). Notice also that the algorithm in Problem 3 gives higher gains and the pole positions farther in the left half plane.

4.2. Decentralized control of a platoon of vehicles

A feedback-linearized state space model of a platoon of \( N \) automotive vehicles is based, according to Stanković et al. (2000), on the following feedback linearized individual vehicle model:

\[
\dot{x}_i = u_i - d_i + \tau_i^{-1} a_i + \tau_i^{-1} u_i, \quad (30)
\]

where \( d_i = x_{i-1} - x_i \) is the distance between two consecutive vehicles, \( x_{i-1} \) and \( x_i \) being their positions, \( v_i \) and \( a_i \) are the velocity and acceleration of \( i \)th vehicle, respectively, \( u_i \) the input signal chosen to make the closed-loop system satisfy certain performance criteria, and \( \tau_i \) the time constant of the engine. After obtaining the overall platoon state space model with the state \( X = (d_1 - d_r, v_1 - v_r, a_1 - a_r, \ldots, d_N - d_r, v_N - v_r, a_N - a_r)^T \) and input \( u = (u_1, u_2, \ldots, u_N)^T \), where \( d_r, v_r \) and \( a_r \) are the reference values for inter-vehicle distance, velocity and acceleration, respectively, and applying the state and input expansion by using convenient full-rank linear transformations, the following model in the expanded space is obtained (Stanković et al., 2000):

\[
\dot{\tilde{\xi}} = A\tilde{\xi} + B\tilde{\zeta},
\]

(31)

where \( \tilde{\xi} = (\xi_1, \ldots, \xi_N)^T \), \( \xi_i = (v_i, a_i)^T \), \( A = \text{diag}(A_1, \ldots, A_N) \) and \( B = \text{diag}(B_1, \ldots, B_N) \); vectors \( \tilde{\xi} \) and \( \tilde{\zeta} \) and matrices \( A_i \) and \( B_i \) are defined within the formally defined subsystem models connected to each vehicle:

\[
S_i : \dot{\xi}_i = A_i\xi_i + B_i\tilde{\zeta}_i = \begin{bmatrix} A_i^T & 0 \\ -A_d & A_o^T \end{bmatrix} \xi_i + \begin{bmatrix} B_i^T & 0 \\ 0 & B_o^T \end{bmatrix} \tilde{\zeta}_i, \quad (32)
\]

where \( \tilde{\zeta}_i = (v_{i-1} - v_i, a_{i-1} - a_i, d_i - d_r, v_i - v_r, a_i - a_r)^T \) is the state vector of \( i \)th subsystem, \( \xi_i = (v_{i-1}, u_i)^T \) represents its control vector, while \( A_i^T = \begin{bmatrix} 0 & 1 \\ 0 & \tau_i^{-1} \end{bmatrix}, A_d^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B_i^T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \) and \( B_o^T = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \) (Stanković et al., 2000). A decentralized dynamic control law can now be designed for the expanded system using the methodology from Section 3.2, supposing that only the subsystem states \( d_i - d_r, v_i - v_r \) and \( a_i - a_r \) are exactly known in \( i \)th vehicle (subsystem), that is, \( v_{i-1} \) and \( a_{i-1} \) are not accessible in \( i \)th vehicle. According to Section 3.2, the reduced-order Luenberger observer for \( \tilde{\zeta}_i = (v_{i-1} - v_r, a_{i-1} - a_r)^T \) is given by

\[
\hat{w}_i = A_i^T w_i + B_i^T u_{i-1} + L_i \tilde{\zeta}_i = A_d^T \hat{w}_i - A_o^T \tilde{\zeta}_i,
\]

(33)

where \( \tilde{\zeta}_i = (d_i - d_r, v_i - v_r, a_i - a_r)^T \). The local control law has the following specific structure:

\[
u_{i-1} = G_i^1 w_i, \quad u_i = G_i^2 w_i + J_i^2 \tilde{\zeta}_i,
\]

(34)

having in mind that \((i - 1)\)st vehicle does not have any information about \( i \)th vehicle. Matrices \( K_i = \begin{bmatrix} G_i^1 & 0 \\ G_i^2 & -J_i^2 \end{bmatrix} \) and \( L_i, i = 1, \ldots, N \), can now be obtained by using the algorithm from Corollary 1, taking care of the specific lower-block-triangular structure of \( K_i \). For \( \tau_i = \tau = 0.1 \), one obtains: \( G_i = G = [-38.6940, -21.2244], G_i^1 = G_i^2 = [-38.6940, -21.2244], G_i^2 = [0.0095, 0.0005], J_i^2 = J^2 = [351.4028, -319.3970, -13.2356], L_i = L = 10^4 \begin{bmatrix} 0.0001 & 0 & 0 \\ 3.2068 & 0 & 0 \end{bmatrix} \) and \( \tau_i = \tau = 1/4.080 \), with closed loop poles \( 10^2\{-1.1480, -0.0116, -0.1561 + j0.1197, -0.1561 - j0.1197, -0.2640, -320.68, 0\} \). Obviously, it is also possible to apply the alternative design schemes from Section 3.1.

The obtained controller has to be finally contracted to the original space for implementation by using the expansion/contraction matrices as in Šiljak (1991) and Stanković et al. (2000).

5. Conclusion

This paper proposes several schemes for the design of decentralized robust dynamic output controllers for linear interconnected systems with unknown nonlinear interconnections satisfying quadratic constraints. The procedures assume the controller structure containing observers of Luenberger-type, and are in the form of convex optimization problems with LMI constraints giving both feedback and observer gains of local linear dynamic controllers with no explicit constraint on the system model. It is shown that the proposed methodology can be also applied to the case of known linear interconnections, taking advantage of the additional \textit{a priori} knowledge. Two numerical examples illustrate applicability aspects of the proposed procedures. The first example, dealing with inverted pendulums, shows how connective stability can be achieved for large unpredictable variations of system parameters. The second example is especially important, since it shows the applicability of the method to decentralized control of platoons of vehicles, in the case when measurements of the velocity and acceleration of the neighboring vehicles are not available.

Our future goal is to apply the proposed procedures to the design of overlapping decentralized dynamic output controllers for general large-scale systems, within the framework of the Inclusion Principle (Šiljak, 1991; Stanković et al., 2000; Chu & Šiljak, 2005).

References


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