MOSFET DC Models

• In this set of notes we will
  – summarize MOSFET $V_{th}$ model discussed earlier
  – obtain BSIM MOSFET $V_{th}$ model
  – describe $V_{th}$ model parameters used in BSIM
  – develop piece-wise compact MOSFET $I_{DS}$ models:
    ♦ basic equations
    ♦ BSIM equations
  – describe IDS model parameters used in BSIM
  – develop substrate current $I_{sub}$ models to characterize device degradation due to high field effects.
BSIM $V_{th}$ Model

We know that for uniformly doped substrate, $N_{sub}$, the long channel threshold voltage is:

$$V_{th} = V_{TH0} + \gamma \left( \sqrt{\phi_s - V_{BS}} - \sqrt{\phi_s} \right)$$  \hspace{1cm} (1)

where $V_{TH0} \equiv V_{th} \left( @ V_{BS} \right) = 0$

$$\gamma = \frac{\sqrt{2q\varepsilon_{Si}N_{sub}}}{C_{ox}} = \text{body effect coefficient}$$

A unified expression for $V_{th}$ used to model the non-uniform vertical channel doping profile is:

$$V_{th} = V_{TH0} + K_1 \left( \sqrt{\phi_s - V_{BS}} - \sqrt{\phi_s} \right) - K_2 V_{BS}$$  \hspace{1cm} (2)

where $K_1$ and $K_2$ model the vertically non-uniform doping effect on $V_{th}$. 

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BSIM $V_{th}$ Model

We derived $V_{th}$ model due to non-uniform vertical and lateral doping profile as:

$$V_{th} = V_{TH0} + K_1 \left( \sqrt{\phi_s - V_{BS}} - \sqrt{\phi_s} \right) - K_2 V_{BS} + K_1 \left( \sqrt{1 + \frac{N_{LX}}{L}} - 1 \right) \sqrt{\phi_s}$$

(3)

where

- $V_{TH0}$ is a model parameter extracted from the measure $I_{DS}$ vs. $V_{GS}$ data at $V_{BS} = 0$
- $K_1$ and $K_2$ models the effect of non-uniform vertical channel doping profile on $V_{th}$ and are fitting parameter extracted from the measured data
- $N_{LX}$ models the non-uniform lateral profile on $V_{th}$ and is a fitting parameter extracted from the measured data.
Modeling SCE due to DIBL

By solving Poisson Eq along the channel, $V_{th}$ shift due to DIBL can be shown as:

$$\Delta V_{th} = \theta_{th}(L)[2(V_{bi} - \phi_s) + V_{DS}]$$

(4)

$V_{bi} = \text{built-in voltage of the S/D junctions given by:}$

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_{CH}N_{DS}}{n_i^2}\right)$$

(5)

where

- $N_{DS} = \text{source-drain doping concentration}$
- $N_{CH} = \text{channel doping concentration}$

Eq (4) shows that $\Delta V_{th}$ depends linearly with $V_{DS}$. And, $V_{th}$ decreases as $V_{DS}$ increases due to DIBL.
Modeling SCE due to DIBL

Now, \( \theta_{th}(L) = e^{-\frac{L}{2l_t}} + 2e^{-\frac{L}{l_t}} \) \hspace{1cm} (15)

where \( l_t \) is a characteristic length given by:

\[
\begin{align*}
l_t &= \sqrt{\frac{\varepsilon_{Si} T_{OX} X_{dep}}{\varepsilon_{ox} \eta}} \\
&\text{with } X_{dep} = \sqrt{\frac{2\varepsilon_{Si} (\phi_s - V_{BS})}{qN_{CH}}}
\end{align*}
\]

\( \eta \) = fitting parameter accounts for the approximations used to obtain \( l_t \).

In order to account for non-uniform vertical channel doping concentration, (15) is modified so that from (13):

\[
\Delta V_{thd} = D_{VT0} \left( e^{-D_{VT1} \frac{L_{eff}}{2l_t}} + 2e^{-D_{VT1} \frac{L_{eff}}{l_t}} \right) (V_{bi} - \phi_s) \\
+ \left( e^{-D_{SUB} \frac{L_{eff}}{2l_{t0}}} + 2e^{-D_{SUB} \frac{L_{eff}}{l_{t0}}} \right) \left( E_{TA0} + E_{TAB} V_{BS} \right) V_{DS}
\] \hspace{1cm} (16)
Narrow Width Effect - An Empirical Model

$V_{th}$ shift due to narrow width effect can be shown as:

$$\Delta V_{thW} \propto \frac{T_{OX}}{W_{eff}} \phi_s$$  \hspace{1cm} (17)

To model both narrow width and reverse narrow width effects, the model is expressed in terms of fitting parameters: $K_3$, $K_{3B}$, and $W_0$ to get:

$$\Delta V_{thW} = (K_3 + K_{3B} V_{BS}) \frac{T_{OX}}{W_{eff} + W_0} \phi_s$$  \hspace{1cm} (18)

where $W'_{eff} = \text{effective channel width}$.

Introducing SCE in narrow devices, we add to (18):

$$\Delta V_{thWL} = D_{VT0W} \left( -D_{VT1W} \frac{W'_{eff} L_{eff}}{2l_{tw}} + 2e^{-D_{VT1W} \frac{W'_{eff} L_{eff}}{l_{tw}}} \right) (V_{bi} - \phi_s)$$  \hspace{1cm} (19)
Complete $V_{th}$ Model

\[
V_{th} = V_{TH0} + K_1 \left( \sqrt{\phi_s - V_{bs}} - \sqrt{\phi_s} \right) - K_2 V_{bs} + K_1 \left( \sqrt{1 + \frac{N_{LX}}{L_{eff}} - 1} \right) \sqrt{\phi_s} + \left( K_3 + K_{3B} V_{bs} \right) \frac{T_{OX}}{W_{eff}} \phi_s
\]

\[-D_{VT0} \left( e^{-D_{VT1} \frac{L_{eff}}{2l_t}} + 2e^{-D_{VT1} \frac{L_{eff}}{l_t}} \right) (V_{bi} - \phi_s) - \left( e^{-D_{SUB} \frac{L_{eff}}{2l_{t0}}} + 2e^{-D_{SUB} \frac{L_{eff}}{l_t}} \right) (E_{TA0} + E_{TAB} V_{BS}) V_{DS}
\]

\[-D_{VT0W} \left( e^{-D_{VT1W} \frac{W_{eff} L_{eff}}{2l_{tw}}} + 2e^{-D_{VT1W} \frac{W_{eff} L_{eff}}{l_{tw}}} \right) (V_{bi} - \phi_s)
\]

(30)

“1” ⇒ vertical non-uniform channel doping effect
“2” ⇒ lateral non-uniform channel doping effect
“3” ⇒ narrow width effect
“4” ⇒ SCE due to DIBL

Small $L, W$ effect
## $V_{th}$ Model Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{OX}$</td>
<td>gate oxide thickness</td>
</tr>
<tr>
<td>$T_{OXM}$</td>
<td>nominal $T_{OX}$ at which parameters are extracted</td>
</tr>
<tr>
<td>$X_{J}$</td>
<td>junction depth</td>
</tr>
<tr>
<td>$N_{CH}$</td>
<td>channel doping concentration</td>
</tr>
<tr>
<td>$N_{SUB}$</td>
<td>substrate doping concentration</td>
</tr>
<tr>
<td>$V_{TH0}$</td>
<td>threshold voltage @ $V_{bs} = 0$ for large $L$</td>
</tr>
<tr>
<td>$V_{FB}$</td>
<td>Flat band voltage</td>
</tr>
<tr>
<td>$K_{1}$</td>
<td>first-order body effect coefficient</td>
</tr>
<tr>
<td>$K_{2}$</td>
<td>second-order body effect coefficient</td>
</tr>
<tr>
<td>$K_{3}$</td>
<td>narrow width coefficient</td>
</tr>
<tr>
<td>$K_{3B}$</td>
<td>body effect coefficient of $K_{3}$</td>
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</table>
### $V_{th}$ Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_0$</td>
<td>narrow width parameter</td>
</tr>
<tr>
<td>$N_{LX}$</td>
<td>lateral non-uniform doping coefficient</td>
</tr>
<tr>
<td>$D_{VT0W}$</td>
<td>first coefficient of narrow width effect on $V_{th}$ at small $L$</td>
</tr>
<tr>
<td>$D_{VT1W}$</td>
<td>second coefficient of narrow width effect on $V_{th}$ at small $L$</td>
</tr>
<tr>
<td>$D_{VT2W}$</td>
<td>body-bias coefficient of narrow width effect on $V_{th}$ at small $L$</td>
</tr>
<tr>
<td>$D_{VT0}$</td>
<td>first coefficient of SCE on $V_{th}$</td>
</tr>
<tr>
<td>$D_{VT1}$</td>
<td>second coefficient of SCE on $V_{th}$</td>
</tr>
<tr>
<td>$D_{VT2}$</td>
<td>body-bias coefficient of SCE on $V_{th}$</td>
</tr>
<tr>
<td>$V_{BM}$</td>
<td>maximum applied body bias in $V_{th}$ calculation</td>
</tr>
</tbody>
</table>
Piece-Wise $I_{DS}$ Model: Inversion Layer Conductance

Let us apply a large potential at the gate of an MOS capacitor to cause inversion, i.e. $V_G > V_{th}$.

The conductance, $1/R_I$ of the inversion layer is:

$$g_I = \frac{1}{R_I} = \frac{W}{L} \int_0^{x_I} q \mu_n n_I(x) dx$$

(1)

Where,

$n_I(x) = e$- density in the inversion layer

$x_I = \text{depth of the inversion layer}$

$\mu_n = e$- mobility in the inversion layer
Inversion Layer Conductance

The inversion layer mobility, referred to as the surface mobility, $\mu_s \approx 1/2$ of bulk mobility. Now, the inversion layer charge per unit area is given by:

$$Q_I = -\int_0^{x_i} q n_I(x)dx$$  \hspace{1cm} (2)

Assuming $\mu_s = \text{constant}$, we get from (1) and (2):

$$g_I = \frac{W}{L} \mu_s Q_I \quad (- \Rightarrow e^-)$$  \hspace{1cm} (3)

The inversion layer resistance of an elemental length $dy$ is:

$$dR_I = \frac{dy}{W \mu_s Q_I}$$  \hspace{1cm} (4)

Eq. (4) can be used to derive drain current for MOSFETs under appropriate biasing conditions.
Let us consider the following MOSFET structure. The gate bias $V_G$ provides the control of surface carrier densities.

If $V_G > V_{th}$, an inversion layer exists. ∴ a conducting channel exists from D $\rightarrow$ S and current $I_D$ will flow. $V_{th}$ is determined by the properties of the structure and is given by:

$$V_{th} = \phi_{MS} - \frac{Q_f}{C_{ox}} + 2\phi_F + \frac{\sqrt{2qN_{SUB}\varepsilon_{Si}(2\phi_F)}}{C_{ox}} \quad (5)$$

For $V_G < V_{th}$, the structure consists of two diodes back to back and only leakage current flows ($\approx I_o$ of PN junctions); i.e., $I_D \sim 0$. 

Strong Inversion Region
Note:

- The depletion region is wider around the drain because of the applied drain voltage $V_D$.
- The potential along the channel varies from $V_D$ at $y = L$ to 0 at $y = 0$ between the drain and source.
- The channel charge $Q_I$ and the bulk charge $Q_b$ will in general be $f(y)$ because of the influence of $V_D$, i.e. potential varies along the L only $\Rightarrow$ Gradual channel approximation.
Drain Current Model

From (4), the voltage drop across an elemental length $dy$ in the MOSFET channel is:

$$dV = I_D dR_I = \frac{I_D dy}{W \mu_n Q_I(y)} \tag{6}$$

Now, at any point in silicon the induced charge due to $V_G$ is:

$$Q_s(y) = Q_i(y) + Q_B(y) \tag{7}$$

Again, $V_G = V_{FB} - Q_s/C_{OX} + \phi_s \tag{8}$

Here $V_{FB}$ includes $\phi_{MS}$ and $Q_f$. Combining (7) and (8):

$$Q_i(y) = -\{V_G - V_{FB} - \phi_s(y)\}C_{OX} - Q_B(y) \tag{9}$$

Since the surface is inverted, $\phi_s \approx 2\phi_B \ (\equiv 2\phi_F) \text{ plus any reverse bias between the channel and substrate (due to } V_D \text{ or } V_{BS}). \therefore \phi_s(y) = V(y) + 2\phi_B \tag{10}$
Drain Current Model

Also, we know from MOS capacitor analysis that the bulk or depletion charge at any point \( y \) in the channel is:

\[
Q_b(y) = -qN_{\text{SUB}}X_{\text{dep}}(y) = -\sqrt{2q\varepsilon_{\text{Si}}N_{\text{SUB}}[V(y) + 2\phi_B]} \tag{11}
\]

Note that as we move from \( S \rightarrow D \), \( V(y) \uparrow \) due to \( IR \) drop in the channel.

\[\therefore X_{\text{dep}} \uparrow \text{ as we move toward the drain and} \]

\[Q_{b} \uparrow \text{ as we move toward the drain.} \]

Substituting (10) and (11) into (9), we get the expression for inversion charge:

\[
Q_i(y) = -\{V_G - V_{\text{FB}} - V(y) - 2\phi_B\}C_{\text{OX}}
+ \sqrt{2q\varepsilon_{\text{Si}}N_{\text{SUB}}[V(y) + 2\phi_B]} \tag{12}
\]
Basic Drain Current Model

Substituting (12) in (6), we get:

$$I_D dy = -W \mu_s \left( \left[ V_G - V_{FB} - V(y) - 2 \phi_B \right] C_{ox} - \sqrt{2q \varepsilon_{Si} N_{SUB} [V(y) + 2 \phi_B]} \right) dV$$

If we assume $Q_b(y) = \text{constant}$, i.e. neglect the influence of channel voltage on $Q_b$, then:

$$I_D dy = -W \mu_s C_{ox} \left( V_G - V_{FB} - V(y) - 2 \phi_B - \frac{\sqrt{2q \varepsilon_{Si} N_{SUB} [V(y) + 2 \phi_B]}}{C_{OX}} \right) dV$$

$$= -W \mu_s C_{ox} \left[ V_G - V_{th} - V(y) \right] dV$$

(13)

In (13) $V_{th}$ includes the effects of $\phi_B$, $\phi_{MS}$, and $Q_b(y = 0)$.

Integrating within the limits: $y = 0$ to $y = L$:

$$I_D = \frac{W}{L} \mu_s C_{ox} \left[ V_G - V_{th} - \frac{V_D}{2} \right] V_D$$

(14)
Basic Drain Current Model: Linear Region

Eq. (14) is the Level 1 MOSFET $I_{DS}$ model. If we define,

\[
\kappa \equiv \mu_s C_{ox} = \text{process transconductance}
\]
\[
\beta \equiv \mu_s C_{ox} (W/L) = \kappa (W/L) = \text{gain factor of the device}
\]

\[
I_D = \beta [V_G - V_{th} - \frac{V_D}{2}] V_D
\]

(15)

If $V_D < 0.1 \text{ V}$, we can simplify (15) to:

\[
I_D \approx \beta (V_G - V_{th}) V_D
\]

(16)

(16) shows that current varies linearly with $V_D$. This is defined as the \textit{linear region} of operation. From (16), the effective resistance between the source and drain is:

\[
R_{ch} = \frac{V_D}{I_D} \approx \frac{1}{\beta (V_G - V_{th})}
\]

(17)
Basic Drain Current Model: Linear Region

- $I_D$ Vs. $V_D$ from (15) with different $V_G$ show $I_D \downarrow$ for higher $V_D$. While the measured $I_D$ saturates at higher $V_D$. This discrepancy is due to the breakdown of gradual channel approximation near the drain–end of the channel at high $V_D$.

- Eq. (15) is valid only as long as an inversion layer exists all the way from $S \rightarrow D$.

- The maximum value of $I_D$ vs. $V_D$ plots occurs at:
  \[ V_D = V_G - V_{th} \equiv V_{DSAT} = \text{drain saturation voltage} \quad (18) \]

- For $V_D > V_{DSAT}$, a channel will not exist all the way to the drain.
Basic Drain Current Model: Saturation
Basic Drain Current Model: Saturation

- When $V_D > V_{DSAT}$,
  - $e$-travelling in the inversion layer are injected into the pinch-off region near the drain-end.
  
  - The high $\varepsilon$-field in the pinch-off region pulls $e$- into the drain.
  
  - Further increase of $V_D$ do not change $I_D$ (to a first order).
    
    $\therefore I_D \approx \text{constant for } V_D > V_{DSAT}$
  
  - The boundary between the linear and the saturation regions is described by: $V_G - V_{th} = V_{DSAT}$.
Basic Drain Current Model: Saturation

Substituting (18) in (16), we can show that the saturation drain current is given by:

\[
I_{DSAT} \approx \frac{W}{2L} \mu_n C_{OX} (V_G - V_{th})^2
\]  

(19)

⇒ square law theory of MOSFET devices.

At \( V_D > V_{DSAT} \), channel is pinched-off and the effective channel length is given by: \( L_{eff} = L - l_d = L(1 - l_d/L) \)

\[
\therefore I_D \approx \frac{W}{2(L - l_d)} \mu_n C_{OX} (V_G - V_{th})^2 = \frac{I_{DSAT}}{\left(1 - \frac{l_d}{L}\right)}
\]  

(20)

Typically, \( l_d << L \), then from (20):

\[
I_D \approx I_{DSAT} \left(1 + \frac{l_d}{L}\right)
\]  

(21)
Basic Drain Current Model: Saturation

Since $I_d$ depends on $(V_D - V_{DSAT})$, we can write:

$$1 + \frac{l_d}{L} \equiv 1 + \lambda (V_D - V_{DSAT})$$

(22)

Here, $\lambda$ = channel length modulation (CLM) parameter.

$$\therefore I_D \approx I_{DSAT} \left(1 + \lambda (V_D - V_{DSAT})\right)$$

(23)

From (23), $I_D = 0$ at $(V_D - V_{DSAT}) = -1/\lambda$.

$$\therefore \lambda = \text{Early voltage due to CLM}$$

$\lambda$ can be extracted from the $V_D$-intercept of $I_D$ vs. $V_D$ plots at $I_D = 0$ as shown in Fig.

$\frac{1}{\lambda} \equiv V_{ACLM}$ = Early voltage due to CLM
Basic Drain Current Model: Summary

- **Current Eq:**

\[
I_D = \begin{cases} 
0 & V_G \leq V_{th} \quad \text{(cut-off region)} \\
\beta [V_G - V_{th} - \frac{V_D}{2}] V_D & V_G \geq V_{th}, V_D \leq V_{DSAT} \quad \text{(linear region)} \\
\frac{\beta}{2} (V_G - V_{th})^2 (1 + \lambda V_D) & V_G \geq V_{th}, V_D \geq V_{DSAT} \quad \text{(saturation region)}
\end{cases}
\]

(24)

- **Model Parameters:**
  - \(V_{TO}\) = threshold voltage at \(V_B = 0\)
  - \(K_P\) = process transconductance
  - \(\Gamma\) = body factor
  - \(\Lambda\) = channel length modulation factor
  - \(\Phi\) = \(2|\phi_B|\) = bulk Fermi-potential.
Basic Drain Current Model: Summary

• Assumptions:
  – gradual channel approximation (GCA) is valid
  – majority carrier current is negligibly (such as hole current for nMOSFETs is neglected)
  – recombination and generation are negligible
  – current flows in the y-direction (along the length of the channel) only
  – carrier mobility $\mu_s$ in the inversion layer is constant in the y-direction
  – current flow is due to drift only (no diffusion current)
  – bulk charge $Q_b$ is constant at any point in the y-direction.

*The accuracy of The basic model is poor even for long channel devices.*
Bulk Charge Effect

In reality, $Q_b$ varies along the channel from the source at $y = 0$ to drain at $y = L$ because of the applied bias $V_D$. Then from (11) with back bias $V_{BS}$ we have:

$$Q_b(y) = -qN_{SUB}X_{dep}(y) = -\sqrt{2q\varepsilon_{Si}N_{SUB}}[V(y) + 2\phi_B + V_{BS}]$$

$$= -C_{ox}\gamma \sqrt{2\phi_B + V_{BS} + V(y)} \quad (25)$$

$$\therefore Q_I(y) = -C_{ox} \left[ V_G - V_{FB} - 2\phi_B - V(y) - \gamma \sqrt{2\phi_B + V_{BS} + V(y)} \right] \quad (26)$$

For the simplicity of computation, we simplify (25) by Taylor series expansion and by neglecting higher order terms, we get:

$$Q_b(y) = C_{ox}\gamma \left[ \sqrt{(2\phi_B + V_{BS})} + \frac{1}{2} \frac{V(y)}{\sqrt{(2\phi_B + V_{BS})}} - \ldots \right]$$

$$\simeq C_{ox}\gamma \left[ \sqrt{(2\phi_B + V_{BS})} + \frac{0.5}{\sqrt{(2\phi_B + V_{BS})}} V(y) \right] = C_{ox}\gamma \left[ \sqrt{2\phi_B + V_{BS}} + \delta V(y) \right] \quad (27)$$
Bulk Charge Effect

where \( \delta \approx \frac{0.5}{\sqrt{2\phi_F + V_B}} \)

From (26) and (27),

\[
Q_I(y) = -C_{ox}(V_G - V_{FB} - V(y) - 2\phi_B) + Q_b(y) \\
= -C_{ox}(V_G - V_{FB} - V(y) - 2\phi_B - \gamma \sqrt{2\phi_F + V_{BS} + V(y)}) \\
= -C_{ox}(V_G - V_{FB} - V(y) - 2\phi_B - \gamma \sqrt{2\phi_F + V_{BS} + \delta V(y)}) \\
= -C_{ox}(V_G - V_{FB} - 2\phi_F - \gamma \sqrt{2\phi_F + V_{BS}} - (1 + \delta \gamma) V(y)) \\
= -C_{ox}(V_G - V_{th} - \alpha V(y))
\]

where \( \alpha \equiv \text{bulk charge factor} \approx 1 + \frac{0.5\gamma}{\sqrt{2\phi_F + V_{BS}}} \)

We will use (28) to develop advanced \( I_{DS} \) models.
MOSFET $I_{DS}$ Model with Bulk Charge Effect

- $Q_b$ variation along the channel offers more accurate $I_D$ modeling in the linear and saturation region.

- The simplified MOSFET drain current model using the *bulk-charge factor* $\alpha$ as a fitting parameter is given by:

$$I_D = \begin{cases} 
0 & V_G \leq V_{th} \\
\beta [V_G - V_{th} - \frac{\alpha V_D}{2}]V_D & V_G \geq V_{th}, V_D \leq V_{DSAT} \\
\frac{\beta}{2\alpha} (V_G - V_{th})^2 (1 + \lambda V_D) & V_G \geq V_{th}, V_D \geq V_{DSAT}
\end{cases}$$

*(cut-off region)*  
*(linear region)*  
*(saturation region)*  

Where $V_{DSAT} = (V_G - V_{th}) / \alpha$  

$$\alpha = 1 + \frac{0.5 \gamma}{\sqrt{2\phi_B + V_{BS}}}$$

*(29)*  

*(30)*  

*(31)*
High-Field Effects in $I_{DS}$

- Surface mobility $\mu_s = \text{constant assumed in MOSFET current expressions is not true under high } V_G \text{ and } V_D$.
- As the vertical field $E_x$ and lateral field $E_y$ increase with increasing $V_G$ and $V_D$, respectively, carriers suffer increased scattering. ∴ $\mu_s = f(E_x, E_y)$
- For the simplicity of $I_D$ calculation an effective mobility defined as the average mobility of carriers is used:

$$\mu_{eff} = \frac{\int_0^{X_{inv}} \mu_s(x, y)n(x, y)dx}{\int_0^{X_{inv}} n(x, y)dx}$$

(32)

$$\therefore I_D = \frac{W}{L} \mu_{eff} \int_0^{V_{DS}} Q_i dV$$

(33)
Effect of High Vertical Electric Fields

In reality, $\mu_s$ is highly reduced by large vertical e-fields. The vertical e-field pulls the inversion layer e- towards the surface causing:

1 more *surface scattering*

2 *coulomb scattering* due to interaction of e- with oxide charges ($Q_f$, $N_{it}$).

Since e-field varies vertically through the inversion layer, the average field in the inversion layer is:

$$ E_{\text{eff}} = \frac{E_{x1} + E_{x2}}{2} \quad (34) $$

where $E_{x1}$ = vertical e-field at the Si-SiO$_2$ interface

$E_{x2}$ = vertical e-field at the channel-depletion layer interface.

From Gauss’ Law: $E_{x1} - E_{x2} = Q_{\text{inv}}/K_s\varepsilon_o$ and $E_{x2} = Q_B/K_s\varepsilon_o$ \quad (35)

:. From (34) & (35) we can show: $E_{\text{eff}} = [(0.5Q_{\text{inv}} + Q_B)]/K_s\varepsilon_o \quad (36)$
Universal Mobility Behavior

- In general,
  \[ E_{\text{eff}} = \frac{(Q_B + \eta Q_{\text{inv}})}{K_s \varepsilon_0} \]
  where for <100> Si
  \( \eta = 1/2 \) for electrons
  \( \eta = 1/3 \) for holes.

- Measured \( \mu_{\text{eff}} \) vs. \( E_{\text{eff}} \)
  plots at low \( V_D \) show:
  - a universal behavior independent of doping concentration at high vertical fields.
  - dependence on: 1) doping concentration and 2) interface charge at low vertical fields.
The experimentally observed *mobility behavior* is due to the relative contributions of different scattering mechanisms set by the strength:

1. **Coulomb scattering** by ionized impurities and oxide charges.
2. **Phonon scattering**
3. **Surface roughness scattering** at the Si-SiO₂ interface.
Universal Mobility Behavior

• *Surface roughness scattering*↑ as carrier confinement close to the interface↑ at high vertical e-fields. ∴ \( \mu_{\text{eff}} \downarrow \) as \( E_{\text{eff}} \uparrow \).

• The experimentally observed *universal behavior* occurs because phonon scattering is weakly dependent on vertical e-fields.

• Deviations from the *universal behavior* occur in heavily doped substrates at *low fields* due to:
  – dominant *ionized impurity scattering* at low inversion charge densities. ∴ \( \mu_{\text{eff}} = f(N_{\text{ch}}) \).
  – *coulomb scattering* by
    • ionized impurities in the depletion region
    • oxide charges.

• *Phonon scattering* has the strongest temperature dependence on \( \mu_{\text{eff}} \).
Effective Mobility due to High $V_G$

Substituting for $Q_b$ from $V_{th}$ Eqn and $Q_I$ in (36), we get:

$$E_{eff} = \frac{V_{GS} + V_{th}}{6T_{OX}}$$  \hspace{2cm} (37)

The dependence of $\mu_{eff}$ on $V_G$ is described by an empirical relation:

$$\mu_{eff} = \frac{\mu_0}{\left(1 + \frac{E_{eff}}{E_0}\right)^\nu}$$  \hspace{2cm} (38)

Where

- $\mu_0 =$ the maximum extracted value of low-field mobility at a given doping concentration $\equiv$ low-field surface mobility.
- $\nu \approx 0.25$ for electrons and $\nu \approx 0.15$ for holes.
- $E_c \approx 2.7 \times 10^4$ V/cm = critical e-field above which $\mu_0 \downarrow$. 


Mobility Degradation due to $V_G$

By Taylor’s series expansion of (38) and introducing $V_{BS}$ dependence, the vertical field mobility degradation model can be shown as:

$$\mu_{\text{eff}} = \frac{\mu_0}{1 + U_a \left( \frac{V_{GS} + V_{th}}{T_{OX}} \right) + U_b \left( \frac{V_{GS} + V_{th}}{T_{OX}} \right)^2}$$

(39)

where $U_a$ and $U_b$ are the model parameters extracted from $I_D$ - $V_G$ characteristics.

The $V_{BS}$ dependence is included by model parameters $U_c$:

$$\mu_{\text{eff}} = \frac{\mu_0}{1 + (U_a + U_c V_{BS}) \left( \frac{V_{GS} + V_{th}}{T_{OX}} \right) + U_b \left( \frac{V_{GS} + V_{th}}{T_{OX}} \right)^2}$$

(40)
Effect of High Lateral Electric Fields

Additional complication arises because of the high lateral e-fields.

We see that for e- in silicon \( v_d \) saturates near \( E \sim 10^4 \) V/cm and \( v_d = \mu E \) does not hold.

Since average e-field for short channel devices > \( 10^4 \) V/cm.

\[ \therefore \text{small geometry MOSFET devices will operate at } v_d = v_{sat} \approx 10^7 \text{ cm/sec.} \]

Since \( \mu \neq \text{constant} \), we must account for high lateral e-field effects in the expression for \( I_D \) derived from simple theory.
Mobility Degradation due to $V_D$

- The velocity saturation of inversion carriers due to increased lateral field $E_y$ causes:
  - current saturation sooner than predicted by $V_{DSAT} = V_G - V_{th}$
  - lower $I_{DSAT}$ than predicted by simple theory.

- The drift velocity due to high field effect is given by:

$$v_d = \frac{\mu_{eff} E_y}{\left[1 + \left(\frac{E_y}{E_c}\right)^\beta\right]^{\frac{1}{\beta}}}$$

for $E > E_c$

Where $E_c = \frac{v_{sat}}{\mu_{eff}} = \text{critical lateral field for velocity saturation}$

$\beta = 2$ for electrons; $\beta = 1$ for holes.

- For simplicity of numerical solution, $\beta = 1$ is used.

$$\therefore v_d = \frac{\mu_{eff} E_y}{1 + \frac{\mu_{eff} E_y}{v_{sat}}}$$

(41)
Mobility Degradation due to $V_G$ and $V_D$

- The effective mobility due to the combine effect of $V_G$ and $V_D$ is given by:

$$\mu_{\text{eff}} = \frac{\mu_0}{1 + \theta(V_G - V_{th}) + \theta_b V_B + \theta_c V_D}$$  \hspace{1cm} (42)

Where

- $\theta_c \equiv 1/LE_C$
- $L =$ channel length of MOSFETs.

- Thus, $\mu_{\text{eff}}$ is modeled by the parameter set:
  - $\{(\mu_0, \theta, \theta_b, \nu_{\text{sat}} (E_C))\}$
  - the model parameters are obtained by curve fitting the experimental data to the model equation.
Assume a piecewise linear model, i.e., $v_d$ saturates abruptly at $E = E_c$:

$$v_d = \frac{\mu_{\text{eff}} E}{1 + \frac{E}{E_c}}, \quad (E \leq E_c)$$

$$= v_{\text{sat}}, \quad (E \geq E_c) \quad (43)$$

Where $E$ = lateral e-field and $E_c$ = critical e-field at which carriers are velocity saturated, i.e. $v = v_{\text{sat}}$.

Now, the current at any point $y$ in the channel is given by:

$$I_D = I(y) = W C_{\text{ox}} [V_G - V_{\text{th}} - \alpha V(y)] v(y) \quad (44)$$

Where

$V(y)$ = potential difference between the drain and channel at $y$.

$v(y)$ = carrier velocity at any point $y$ in the channel.
$I_{DS}$ Model in Strong Inversion

Substituting (43) in (44) we get:

$$E(y) = \frac{I_D}{W \mu_{eff} C_{ox} [V_G - V_{th} - \alpha V(y)] - \frac{I_D}{E_c} = - \frac{dV(y)}{dy}}$$  \hspace{1cm} (45)$$

Integrating (45) from $y = 0$ to $y = L$ with corresponding $V(y) = 0$ to $V(y) = V_D$, we get in the linear region ($V_{DS} \leq V_{DSAT}$) current:

$$I_D = \frac{W \mu_{eff} C_o \left( V_G - V_{th} - \frac{1}{2} \alpha V_{DS} \right) V_{DS}}{L \left( 1 + \frac{V_{DS}}{E_c L} \right)}$$  \hspace{1cm} (46)$$
Let us define, $V_{DSAT} \equiv$ the drain saturation voltage due to $v_{sat}$, i.e. at $E = E_c$.

Using this condition in (45), we get in the saturation region ($V_{DS} \geq V_{DSAT}$):

$$I_D = \frac{W\mu_{eff} C_ox (V_G - V_{th} - \alpha V_{DSAT}) E_c}{2}$$

(47)

Including channel length modulation (CLM):

$$I_D = \frac{W\mu_{eff} C_ox E_c}{2} (V_G - V_{th} - \alpha V_{DSAT}) \left(1 + \frac{V_{DS} - V_{DSAT}}{V_{ACLM}}\right)$$

(48)

where $V_{ACLM} =$ channel length modulation parameter
$I_{DS}$ Model in Strong Inversion

Since $I_D$ given by (46) and (47) must be continuous @ $V_{DS} = V_{DSAT}$, therefore, equating (46) = (47):

$$V_{DSAT} = \frac{E_C L (V_G - V_{th})}{\alpha E_C L + (V_G - V_{th})}$$  \hspace{1cm} (49)

Note that from (43), $v_{sat} = \mu_{eff} E_c/2$. Thus, the drain current models in strong inversion:

$$I_D = \frac{W \mu_{eff} C_o}{L \left(1 + \frac{V_{DS}}{E_c L}\right)} \left(V_G - V_{th} - \frac{1}{2} \alpha V_{DS}\right)V_{DS}, \text{for } V_{GS} > V_{th}, V_{DS} < V_{DSAT}$$

$$I_D = WC_{ox} v_{sat} (V_G - V_{th} - \alpha V_{DSAT}) \left(1 + \frac{V_{DS} - V_{DSAT}}{V_A}\right), \text{for } V_{GS} > V_{th}, V_{DS} > V_{DSAT}$$  \hspace{1cm} (50)
3. Sub-threshold Region Model

Simple long channel device equations assume: $I_{DS} = 0$ for $V_{GS} < V_{th}$. In reality, $I_{DS} \neq 0$ and varies exponentially with $V_{GS}$ in a manner similar to a bipolar transistor.

In order to develop a theory of sub-threshold conduction, let us consider MOS band diagram with applied source (S) and gate (G) bias measured with respect to the substrate (Sub), that is:

- $V_{SSub}$
- $V_{GSub}$
In the sub-threshold (weak inversion) region, we know:

1. As $E_F$ is pulled above $E_i$, the number of minority carrier e\(^-\) at the surface increases exponentially with $E_F - E_i$

2. When $\phi_s = 2\phi_B$, strong inversion ($n_{surf} = p_{bulk}$) is achieved and we obtain $V_{th}$.

3. For $\phi_s < 2\phi_F$, the dominant charges present near the surface are ionized acceptor atoms, i.e. $n_{surf} \ll N_A^-$.
   - Thus, there is no $\varepsilon$-field laterally along the surface since Poisson’s equation is the same everywhere.
   - Thus, any current flow must be due to diffusion only.
Sub-threshold Region Model

\[ I_D = -AqD \frac{dn}{dy} \]  \hspace{1cm} (51)

4 The e- gradient along the channel (dn/dy) must be constant in order to maintain constant current.

\[ I_D = -AqD\left[\frac{n(0) - n(L)}{L}\right] \]  \hspace{1cm} (52)

Now, we can now use carrier statistics to calculate n(0) and n(L). Referring band diagram on page 41:

\[ n(0) = n_i e \frac{q(\phi_s - V_{Sub} - \phi_B)}{kT} \]  \hspace{1cm} (53)

\[ n(L) = n_i e \frac{q(\phi_s - V_{DSub} - \phi_B)}{kT} \]  \hspace{1cm} (54)

Where \( \phi_s \) is the surface potential with respect to the substrate. Also, note that \( \phi_s = \text{constant along the channel } [\varepsilon(y) = 0] \).
Sub-threshold Region Model

Since the charge in the substrate is assumed uniform \((N_{sub})\), then from Poisson’s equation:

\[
\frac{d^2 \phi}{dx^2} = \frac{qN_A}{\varepsilon_s} = -\frac{dE}{dx}
\]  \hspace{1cm} (55)

\(\therefore \phi\) varies parabolically and \(E\) varies linearly with distance.

Since the e- concentration falls off as \(e^{-q\phi/kT}\) away from the surface, essentially, all of the minority carrier e- are contained in a region in which the potential drops by \(kT/q\).

\(\therefore \) The depth of the inversion layer, \(X_{inv} = \Delta \phi/E_s\).

where \(\Delta \phi = kT/q\), and \(E_s\) is the \(\varepsilon\)-field at the surface.

Again, from Gauss’ Law: \(\varepsilon_sE_s = -Q_b = \sqrt{2\varepsilon_s q N_{SUB} \phi_s}\)  \hspace{1cm} (56)

\(\therefore X_{inv} = \frac{\varepsilon_s kT}{\sqrt{2\varepsilon_s q N_{SUB} \phi_s}} = \frac{kT}{q} \sqrt{\frac{\varepsilon_s}{2q N_{SUB} \phi_s}}\)  \hspace{1cm} (57)
Sub-threshold Region Model

Using (53), (54) and (57) in (52), we get:

\[ I_D = \frac{q}{L} W D n_i \left\{ e^{\frac{q(\phi_s - V_{SSub} - \phi_F)}{kT}} - e^{\frac{q(\phi_s - V_{DSub} - \phi_F)}{kT}} \right\} \frac{kT}{q} \sqrt{\frac{\varepsilon_s}{2qN_A \phi_s}} \]  \hfill (58)

Here \( A = WX_{inv} \).

Using \( V_{DSub} = V_{DS} + V_{SSub} \),

\[ I_D = \frac{W}{L} kT D n_i \sqrt{\frac{\varepsilon_s}{2qN_{SUB} \phi_s}} \left\{ e^{\frac{q(\phi_s - V_{SSub} - \phi_F)}{kT}} \right\} \left\{ 1 - e^{\frac{-qV_{DS}}{kT}} \right\} \]  \hfill (59)

To make use of this equation, we need to know how \( \phi_s \) varies with the externally applied potential \( V_G \).

\[ V_{GSub} = V_{FB} + \phi_s + \frac{\sqrt{2q \varepsilon_s N_{SUB} \phi_s}}{C_{ox}} \]  \hfill (60)
Sub-threshold Region Model

Generally, it is more common to use the source potential as the reference so that:

\[ V_{G\text{Sub}} = V_{GS} + V_{SS\text{Sub}} \]
\[ \phi_s = \psi_s + V_{SS\text{Sub}} \]

\[ \therefore I_D = \frac{W}{L} k T \text{D} n_i \sqrt{ \frac{\epsilon_s}{2qN_{SUB}\psi_s} } \left\{ e^{\frac{q(\psi_s - \phi_F)}{kT}} \right\} \left\{ 1 - e^{-\frac{qV_{DS}}{kT}} \right\} \] (61)

\[ V_{GS} = V_{FB} + \psi_s + \frac{\sqrt{2q\epsilon_s N_{SUB}(\psi_s + V_{SS\text{Sub}})}}{C_{ox}} \] (62)

The depletion layer capacitance is:

\[ C_D = \sqrt{\frac{q\epsilon_s N_{SUB}}{2(\psi_s + V_{SS\text{Sub}})}} \] (63)

Therefore, from (62):

\[ \frac{dV_{GS}}{d\psi_s} = 1 + \frac{1}{C_{ox}} \sqrt{\frac{q\epsilon_s N_A}{2(\psi_s + V_{SS\text{Sub}})}} = 1 + \frac{C_D}{C_{ox}} \equiv n \] (64)
In order to eliminate $\psi_s$ from (61) we expand $V_{GS}$ in a series about the point $\psi_s = 1.5\phi_F$ (weak inversion corresponds to $\phi_F \leq \psi_s \leq 2\phi_F$).

$$V_{GS} \cong V_{GS}\bigg|_{\psi_s=1.5\phi_F} + n(\psi_s - 1.5\phi_F)$$

(64)

Where $V_{GS}^* \equiv V_{GS}\bigg|_{\psi_s=1.5\phi_F}$ and is obtained from (62).

Combining (64) with (61) to eliminate $\psi_s$ in the exponential and using (63) to eliminate the square root of $\psi_s$ in (61), we obtain:

$$I_D = \frac{W}{L} \left( \frac{kT}{q} \right)^2 \mu C_D n_i \left\{ e^{-\left[ \frac{q(V_{GS}-V_{GS}^*)}{nkT} + \frac{q\phi_F}{2kT} \right]} \right\} \left\{ 1 - e^{-\frac{qV_{DS}}{kT}} \right\}$$

(65)
Sub-threshold Region Model

Thus, the sub-threshold current is given by:

\[ I_D = \frac{W}{L} \left( \frac{kT}{q} \right)^2 \mu C_D n_i \left\{ \exp \left[ \frac{q(V_{GS} - V_{GS}^*)}{nkT} \right] \right\} \left\{ 1 - e^{-\frac{qV_{DS}}{kT}} \right\} \]

From (47) we note that:

- \( I_D \) depends on \( V_{DS} \) only for small \( V_{DS} \), i.e. \( V_{DS} \leq 3kT/q \), since \( \exp[-qV_{DS}/kT] \to 0 \) for larger \( V_{DS} \).

- \( I_D \) depends exponentially on \( V_{GS} \) but with an “ideality factor” \( n > 1 \). Thus, the slope is poorer than a BJT but approaches to that of a BJT in the limit \( n \to 1 \).

- \( N_{SUB} \) and \( V_{SSub} \) enter through \( C_D \).
In order to change $I_D$ by one decade, we get from (47):

$$\frac{q \partial V_{GS}}{nkT} = \ln 10 \Rightarrow S = \frac{kT}{q} \ln 10(n) = \frac{kT}{q} \ln 10 \left(1 + \frac{C_D}{C_o}\right)$$

$$\therefore S \approx 60 \frac{mV}{\text{decade}} \left(1 + \frac{C_D}{C_o}\right) \quad (@ \text{ room } T) \quad \ldots (67)$$
Sub-threshold Model - Final Note

• In weak inversion or subthreshold region, MOS devices have exponential characteristics but are less “efficient” than BJTs because $n > 1$.

• Subthreshold slope $S$ does not scale and is $\approx$ constant. Therefore, $V_{th}$ can not be scaled as required by the ideal scaling laws.

• $V_{DS}$ affects $V_{th}$ as well as subthreshold currents.

• In order to optimize $S$, the desirable parameters are:
  – thin oxide
  – low $N_A$
  – high $V_{Sub}$. 
Sub-threshold Region Model

The sub-threshold current Eq. used in BSIM model is:

\[ I_{DS} = I_{on} \left( 1 - \exp\left( -\frac{qV_{DS}}{nkT} \right) \right) \exp\left( \frac{q(V_G - V_{th} - V_{off})}{nkT} \right), \text{ for } V_{GS} < V_{th} \]  

(68)

where \( V_{off} = \) offset voltage is a model parameter.

Thus, the piece-wise drain current models for different regions of MOSFET operations:

\[
I_D = \begin{cases} 
I_{on} \left( 1 - \exp\left( -\frac{qV_{DS}}{nkT} \right) \right) \exp\left( \frac{q(V_G - V_{th} - V_{off})}{nkT} \right), & \text{for } V_{GS} < V_{th} \\
\frac{W\mu_{eff}C_{ox}}{L} \left( V_G - V_{th} - \frac{1}{2} \alpha V_{DS} \right) V_{DS}, & \text{for } V_{GS} > V_{th}, V_{DS} < V_{DSAT} \\
WC_{ox}V_{sat} \left( V_G - V_{th} - \alpha V_{DSAT} \right) \left( 1 + \frac{V_{DS} - V_{DSAT}}{V_A} \right), & \text{for } V_{GS} > V_{th}, V_{DS} > V_{DSAT}
\end{cases}
\]
MOS Threshold Voltage, $V_{th}$ Extraction

$V_{th}$ is obtained by linear extrapolation from the maximum slope to $I_{ds} = 0$ of $I_{ds} - V_{gs}$ plot.

We know, 

$$I_{ds} = \beta \left( V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) V_{ds}$$

Define, 

$$V_{th} \equiv V_{gs} \, @ \, I_{ds} = 0 \text{ and } V_{ds} = 50 \text{ mV}$$

$$\therefore I_{ds} = \beta \left( V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) V_{ds} = 0$$

$$\Rightarrow V_{th} = V_{gs} - \frac{V_{ds}}{2}$$
Substrate Bias Dependence of $V_{th}$

Id – Vg Characteristics of Lg = 50nm NMOSFETS (Leff = 35nm)

Vth(Vbs = 0.0) = 0.32V
Vth(Vbs = -0.5) = 0.39V
Vth(Vbs = -1.0) = 0.46V

Vds = 0.05V
Tox = 1.5nm
Substrate Bias Dependence of $V_{th}$

Substrate bias dependence of $V_{th}$ for uniformly doped substrate is given by:

$$V_{th} = V_{th0} \pm \gamma \left( \sqrt{2\phi_F \pm V_{Sub}} - \sqrt{2\phi_F} \right)$$

where

$$\gamma = \sqrt{\frac{2q N_{sub} K_s \epsilon_o}{C_{ox}}} \equiv \text{body factor}$$

$\gamma$ is obtained from:

$$(V_{th} - V_{th0}) \text{ vs. } \sqrt{2\phi_F + V_{BS}} - \sqrt{2\phi_F}$$

where

slope = $\gamma$

$\gamma$ - intercept = $\gamma \sqrt{2\phi}$
Substrate Bias Dependence of $V_{th}$

Substrate bias dependence of $V_{th}$ for non-uniformly doped substrate is given by the plots:

$$(V_{th} - V_{th0}) \text{ } V_s \cdot \sqrt{2 \phi_F + V_{BS}} - \sqrt{2 \phi_F}$$

where

$\gamma_1(k_1) = \text{body effect due to channel concentration}$

$\gamma_2(k_2) = \text{body effect due to substrate concentration}$

$V_{th}$ is given by:

$$V_{th} = V_{th0} + \gamma_1 \left( \sqrt{2 \phi_F - V_{Sub}} - \sqrt{2 \phi_F} \right) + \gamma_2 \left( \sqrt{2 \phi_F - V_{Sub}} - \sqrt{2 \phi_F} \right)$$
Drain Induced Barrier Lowering (DIBL)

- DIBL is defined as the shift in $V_{th}$ due to $V_{ds}$, especially, in short channel devices. DIBL is defined as:

  1) $\Delta V_{th} \equiv V_{th}(V_{ds-low}) - V_{th}(V_{ds} = V_{dd})$

  2) $\frac{\partial V_{th}}{\partial V_{ds}} \equiv \frac{V_{th}(V_{ds-low}) - V_{th}(V_{ds} = V_{dd})}{(V_{dd} - V_{ds-low})}$

- DIBL is calculated from:
  - $\log(I_{ds})$ Vs. $V_{gs}$ plots at $V_{ds-low} = 50$ mV
  $V_{ds} = V_{dd}$.

*Example*: From Figure we get,

$\Delta V_{th} \approx (0.32 - 0.24) V$

$= 80$ mV
Sub-threshold Slope ($S$)

• $S$ is the inverse of $\log(I_{ds})$ vs. $V_{gs}$ plot at $V_{ds-low}$ and is given by:

$$S \equiv 2.3 \cdot \left[ \frac{dV_{gs}}{d(\log(I_{ds}))} \right]$$

• To extract $S$:
  - extract:
    $$I_{ds1} = I_{ds}(V_{th})$$
    $$I_{ds2} = 2 \text{ dec. below}\ I_{ds1}$$
  - Calculate slope
    $$S = 1/\text{slope}.$$
I_{on} \text{ and } I_{off}

- \( I_{on} \) and \( I_{off} \) can be extracted from \( I_{ds} - V_{gs} \) plot at \( V_{ds} = V_{dd} \).
- \( I_{on} \equiv I_{dsat} \) at
  \[
  V_{ds} = V_{gs} = V_{dd}
  \]
- \( I_{off} \equiv I_{ds} \) at
  \[
  V_{ds} = V_{dd} \\
  V_{gs} = 0
  \]
- **Example**: From Fig.,
  - \( I_{on} \approx 440 \, \mu A/\mu m \)
  - \( I_{off} \approx 3 \, nA/\mu m \)
1) For a silicon MOSFET, considering bulk charge effect in current $I_D$ calculate (a) $V_{TO}$, (b) $\lambda$, (c) $\gamma$, and (d) $\beta$ from the measured data shown in table.

<table>
<thead>
<tr>
<th>VGS (V)</th>
<th>VDS (V)</th>
<th>VBS (V)</th>
<th>ID (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0</td>
<td>536</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>-5</td>
<td>360</td>
</tr>
<tr>
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<td>8</td>
<td>0</td>
<td>644</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>-3</td>
<td>420</td>
</tr>
</tbody>
</table>

2) If $E_{eff} = \left[0.5Q_{inv} + Q_B\right]/\varepsilon_{si}$, where $Q_{inv}$ and $Q_B$ are the inversion charge and bulk/depletion charge under the gate, respectively and $\varepsilon_{si}$ is the dielectric constant of silicon. The dependence of surface mobility, $\mu_s$ on process parameters such as $T_{OX}$, $N_{sub}$ etc. and terminal voltages is lumped in $E_{eff}$. Assume $V_{GS} > V_{th}$ and small $V_{DS}$:

(a) Show that $E_{eff} \approx (V_{GS} + V_{th})/6T_{OX}$.

(b) If the effective mobility is modeled by: $\mu_{eff} = \mu_0/[1 + E_{eff}/E_0]]^\eta$, where $\mu_s = \mu_0@V_{GS} = 0$ and $E_0$ and $\eta$ are parameters determined from the measured data. Use the expression for $E_{eff}$ in part (a) to show that:

$$\mu_{eff} = \frac{\mu_0}{1 + U_a \left(\frac{V_{GS} + V_{th}}{T_{OX}}\right) + U_b \left(\frac{V_{GS} + V_{th}}{T_{OX}}\right)^2}$$

Where $U_a$ and $U_b$ are the model parameters that are determined experimentally.